Modeling Fatigue Crack Nucleation in Al 7075

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MOTIVATION

- Significant part of fatigue life when fatigue cracks microstructurally small, most variability in life occurs during this phase
- Microstructurally small cracks initiate at constituent particles in Al7075
- Crack length not observed to be directly related to particle size
- Microstructure in the vicinity of constituent particles has significant effect
- Variability in microstructurally small crack nucleation and growth related to microstructural variability

GOAL

Predict accurate stress and strain (slip) fields at the grain scale for realistic microstructures subject to cyclic loading.

Key Phenomena that should be captured by the model:

- Geometric effects (grain structure)
- Texture effects (orientations)
- Material hardening
- Particle effects
- Damage accumulation (irreversible slip)

METHODOLOGY

Methodology focuses on the modeling of grain-scale mechanics and includes development of a constitutive model for crystal plasticity and a finite element formulation for polycrystals. The constitutive model is informed by experimental observation and is based on underlying phenomena

Constitutive Model

A crystal elasto-viscoplastic model is employed to capture the response of Al 7075-T651.

Decomposition of the deformation gradient into elastic and plastic parts.

$$\mathbf{F} = {}^e\mathbf{F}\,{}^p\mathbf{F}$$

Velocity gradient given in terms of the slip rate on each slip system.

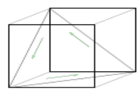
$${}^p\dot{\mathbf{L}} = \sum_\alpha \dot\gamma^\alpha\ \mathbf{P}^\alpha$$

Plastic Slip Model

The crystal plasticity model captures slip-system activity and the interaction of dislocations with precipitates (Orowan looping).

A power law relates rate of shearing on slip systems to resolved shear stress.

$$\dot\gamma^\alpha = \dot\gamma_o \frac{\tau^\alpha}{g^\alpha}\left|\frac{\tau^\alpha}{g^\alpha}\right|^{\frac{1}{m}-1}$$



Evolution of resistance to plastic slip (hardening) is based on the Orowan looping mechanism.[1-3]

$$\dot g^\alpha = G_o\left(\frac{g_s - g^\alpha}{g_s - g_o}\right)\sum_\beta 2\left|S^\alpha_{ij}\, S^\beta_{ij}\right|\left|\dot\gamma^\beta\right|$$

Finite Element Formulation

The finite element implementation allows for the modeling of realistic grain structures. A three-dimensional formulation with additional pressure variable is utilized for stability.

Governing equations:

$$\left(\sigma'_{ij} + p\,\delta_{ij}\right)_{,j} = 0 \qquad \frac{1}{3}\sigma_{ii} - p = 0$$

Corresponding weak forms (total Lagrangian) with interpolation functions:

$$\underbrace{\int_{\Omega_o}\left(\sigma^{h\prime}_{ij} + p^h\,\delta_{ij}\right)\psi_{\alpha,K}F^{-1}_{Kj}\,J\,d\Omega_o}_{f^{int}_{i\alpha}(\mathbf{a},\mathbf{p})} - \underbrace{\int_{\partial\Omega_o} t_i\psi_\alpha\frac{d\Gamma}{d\Gamma_o}d\Gamma_o}_{f^{ext}_{i\alpha}} = 0$$

$$\underbrace{\int_{\Omega_o}\frac{1}{K}\left(\frac{1}{3}\sigma^h_{ii} - p^h\right)\bar\psi_\rho\,J\,d\Omega_o}_{h_\rho(\mathbf{a},\mathbf{p})} = 0$$

Linearized Equations:

$$K^r_{i\alpha j\beta}\,\Delta a_{j\beta} + G^r_{i\alpha\varphi}\,\Delta\bar p_\varphi = f^{ext}_{i\alpha} - f^{int}_{i\alpha}(\mathbf{a}^r,\mathbf{p}^r)$$
$$H^r_{\rho j\beta}\,\Delta a_{j\beta} + M^r_{\rho\varphi}\,\Delta\bar p_\varphi = 0 - h_\rho(\mathbf{a}^r,\mathbf{p}^r)$$

Discontinuous interpolations for $\bar p_\phi$ allow for a $\Delta\bar p$ solution on the element level:

$$\Delta\bar p_\varphi = -M^{r\,-1}_{\varphi\rho}\left(h_\rho(\mathbf{a}^r,\mathbf{p}^r) + H^r_{\rho j\beta}\,\Delta a_{j\beta}\right)$$

IMPLEMENTATION

Constitutive model and finite-element formulation were implemented in C++.

Key features of the formulation:

- Finite strain
- Stable mixed displacement/pressure formulation
- State update routine for elasto-viscoplastic crystal constitutive model
- Consistent tangent formulation for fast convergence

Finite-element driver:

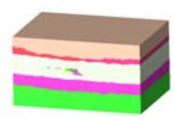
- Utilizes MPICH for parallel processing
- PETSc software package used for solving global system of equations

ABAQUS UMAT:

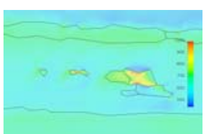
- Alternative to the C++ version of the driver
- Provides a means to compare C++ results to commercial software

RESULTS

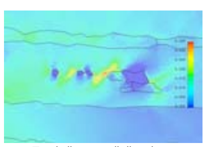
Finite-element analysis of an actual polycrystal of cold rolled Al 7075 with an embedded particle. The model geometry was obtained from experiments run at Northrop Grumman. The grain geometries are extruded through the length of the unit cell, while the particle is only extruded a short distance. The model is used to determine the damage that ultimately lead to the particle cracking.



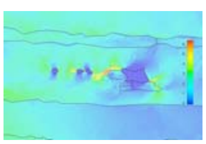
Unit Cell (Particle is the largest gray grain)



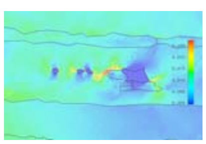
Stress



Total slip over all directions



Damage accumulation



Maximum slip on a plane

Positive Results:

- Consistent with experimental results
- Consistent with results obtained by Cornell using a similar model

Use obtained results and multi-scale analysis to obtain information regarding crack incubation and nucleation for different particles.

SPONSOR

Northrop Grumman

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