$x_1 = \theta \qquad M(\theta) \rightarrow M(x_1)$

$x_2 = \dot{\theta} = \dot{x}_1 \qquad$ Linearize about $\underbrace{(\theta_d, 0)}_{x_1 = \theta_d,\ x_2 = 0}$

$x_1 = \theta - \theta_d \qquad M(\theta) \rightarrow M(x_1 + \theta_d)$

$x_2 = \dot{\theta} = \dot{x}_1 \qquad$ Linearize about $\underbrace{(0, 0)}_{(x_1 = 0,\ x_2 = 0)}$

Linearized system about $\underbrace{(\theta = \theta_d, \dot{\theta} = 0)}$

$\dot{x} = Ax + Bu \rightarrow$ input torques

$y = Cx$

output angles

Create MATLAB LTI object:

$G = ss(A, B, C, \underbrace{0}_{\text{zeros}(2,2)})$

$y \leftarrow \boxed{G} \leftarrow u$

zeros(2, 2): rows, columns

$$G(s) = C(sI-A)^{-1}B$$

$I$: 2x2 identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$sI = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

$$= \frac{C\,\mathrm{Adj}(sI-A)B}{\det(sI-A)}$$

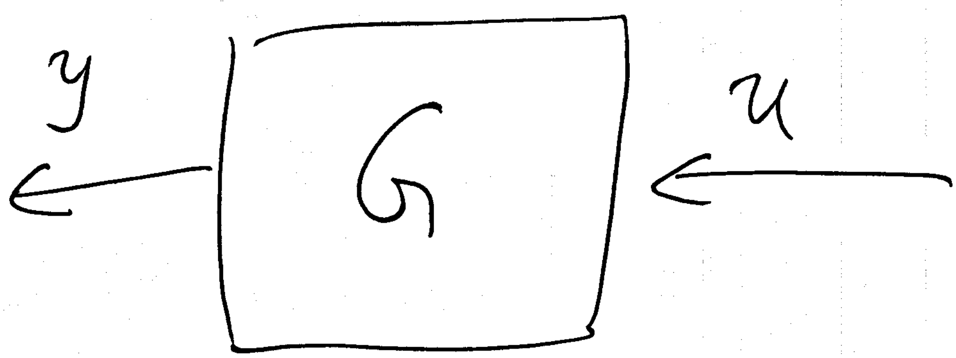
$C\,\mathrm{Adj}(sI-A)B$ ← numerator 2x2 polynomial matrix

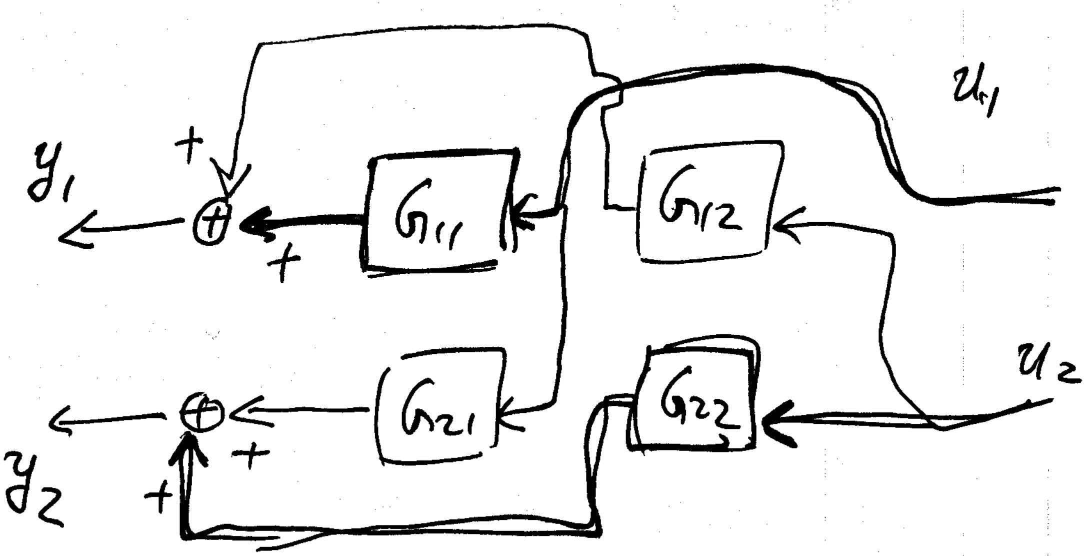
$\det(sI-A)$ ← characteristic polynomial

Gtf = tf(G);

<u>Gtf.num</u>

<u>Gtf.den</u>



## Approximation for SISO design:

$y_1 \leftarrow \boxed{G_{11}} \leftarrow u_1$

$y_2 \leftarrow \boxed{G_{22}} \leftarrow u_2$

Closed loop poles:

pole(feedback(G*K, I))

$2 \times 2$

Use rootlocus + loop shaping to design $K_1, K_2$ for $G_{11}, G_{22}$, respectively.

$\leftarrow \boxed{G} \leftarrow \boxed{\begin{matrix} K_1 & 0 \\ 0 & K_2 \end{matrix}} \leftarrow$

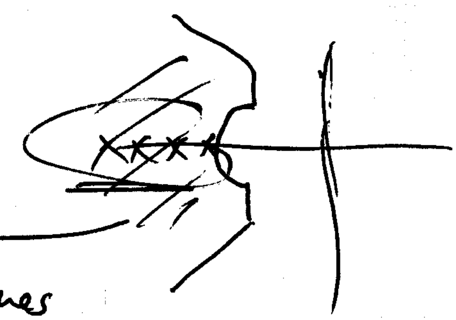
## Direct State Space Design (MIMO technique)

no need to use SISO approximation.

1. Full state feedback $\rightarrow$ Find $F$ such that $\mathrm{eig}(A - BF)$ are at desired locations.

2. Observer design

   $\hookrightarrow$ Find $L$ such that $\mathrm{eig}(A - LC)$ are at desired locations.



5-10 times faster for observer poles.

$F = \text{place}(A, B, \text{desired poles})$

(as a vector)

$L = \text{place}(A^T, C^T, \text{desired observer poles})^T$

# Observer based controller:

$$\overset{\text{estimated state}}{\dot{\hat{x}}} = \underbrace{A\hat{x} + Bu}_{\text{replica of plant}} + \underbrace{L}_{\text{corrective gain}}(\overset{\text{output}}{y} - \overset{\text{estimated output}}{C\hat{x}})$$

$$u = -F\underbrace{\hat{x}}_{\text{replace state by estimated state.}}$$

$$y \longrightarrow \boxed{K} \longrightarrow -u$$

dynamic controller ⇐ LTI object
(observer-based)

state space representation of $K$:

$$\dot{\hat{x}} = A\hat{x} - BF\hat{x} + Ly - LC\hat{x}$$

$$= (A - BF - LC)\hat{x} + Ly$$

$$-u = F\hat{x}$$

$$K = ss(A - B*F - L*C,\ L,\ F,\ \underset{\text{zeros(2,2)}}{0});$$

---

Full state feedback:

$u = -Fx \longrightarrow \dot{x} = (A - BF)x$

$$\begin{bmatrix} \theta_1 - \theta_d \\ \dot{\theta}_1 \\ \theta_2 - \theta_{2d} \\ \dot{\theta}_2 \end{bmatrix}$$

Observer reconstructs $\begin{bmatrix} \theta_1 - \theta_{1d} \\ \boxed{\dot{\theta}_1} \\ \theta_2 - \theta_{2d} \\ \boxed{\dot{\theta}_2} \end{bmatrix}$ from $\begin{bmatrix} \theta_1 - \theta_{1d} \\ \theta_2 - \theta_{2d} \end{bmatrix}$

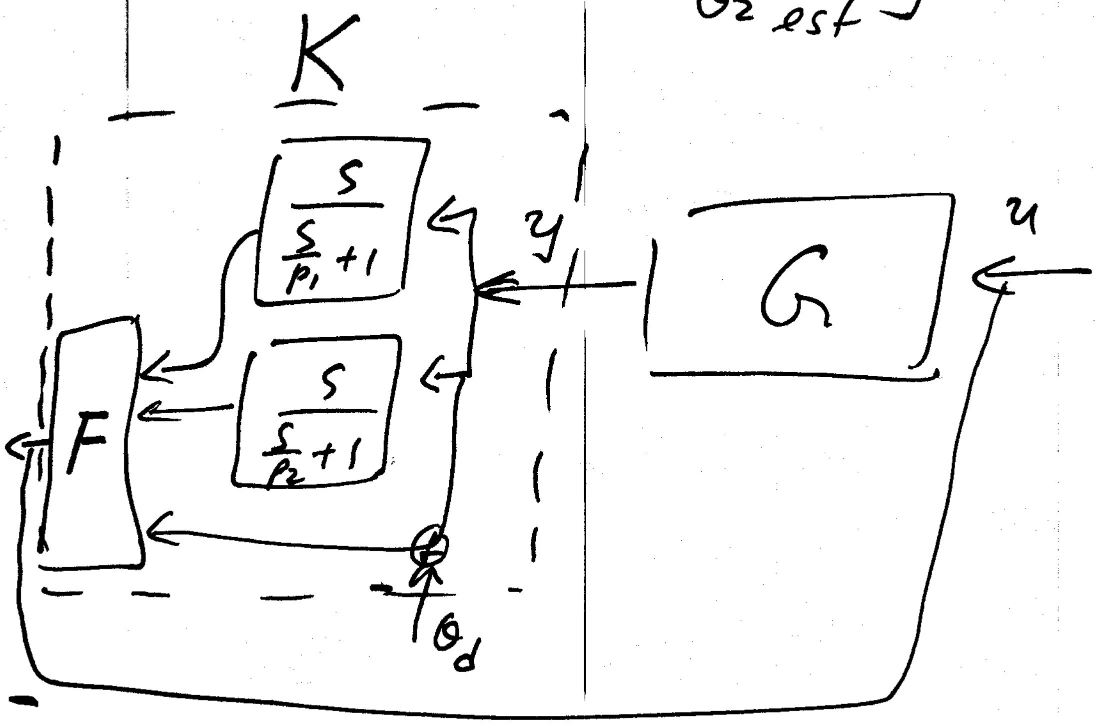
Alternatively:

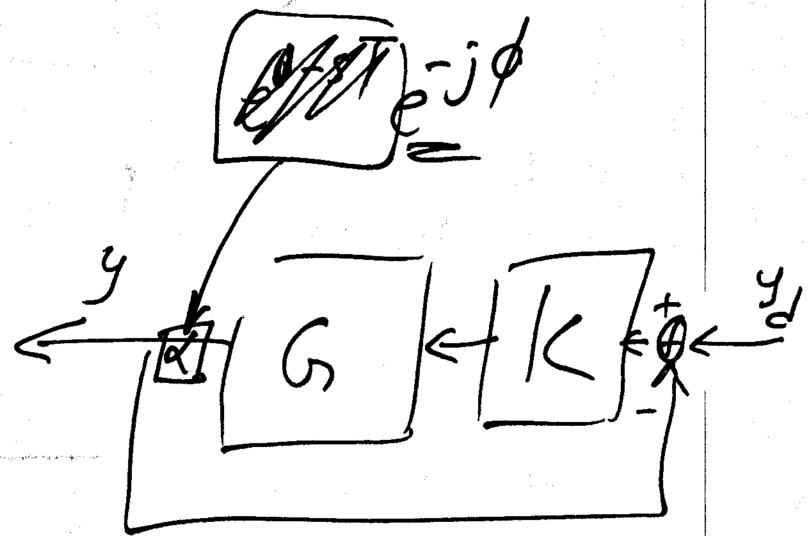
Estimate $\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$ by using washout filters

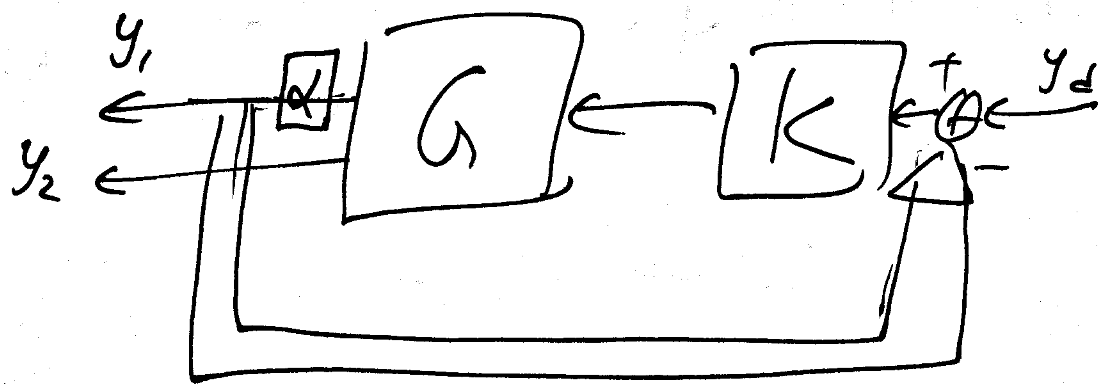
$$\dot{\theta}_{1est} = \underbrace{\frac{s}{\frac{s}{p_1} + 1}}_{\text{washout filter}} \theta_1 \qquad \dot{\theta}_{2est} = \frac{s}{\frac{s}{p_2} + 1} \theta_2$$

Now apply the full state feedback:

$$u = -F\begin{bmatrix} \theta_1 - \theta_{1d} \\ \dot{\theta}_{1\,est} \\ \theta_2 - \theta_{2d} \\ \dot{\theta}_{2\,est} \end{bmatrix}$$






loop gain

$$L_g = \underline{G} * \underline{K};$$

margin(Lg(1,1))   margin(Lg(1,2))

margin(Lg(2,2))   margin(Lg(2,1))

# Discretization of Observer

$$\dot{\hat{x}} = \big(A - L(C - DF) - BF\big)\hat{x} + Ly$$
$$u = -F\hat{x}$$

```
[Ad,Bd]=c2d(A-L*(C-DF)-B*F,L);
```

0.001

'zoh'

'tustin'

```
obsv=c2d(ss(A-L*(C-DF)-B*F,L,-F,0));
[Ad,Bd,Cd,Dd]=ssdata(obsv);
```

$$\hat{x}_{k+1} = A_d\hat{x}_k + B_d y_k$$
$$u_k = -F\hat{x}_k$$

$$\hat{x}_{k+1} = A_d\hat{x}_k + B_d y_k$$
$$u_k = C_d\hat{x}_k + D_d y_k$$

K= ss(A-LC-BF,L,F,0);

Kd= c2d(K,0.001);