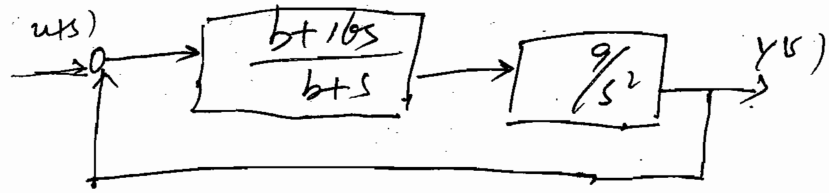
1. 

$$\frac{Y(s)}{R(s)} = \frac{\frac{b+16s}{b+s}\cdot\frac{9}{s^2}}{1+\frac{b+16s}{b+s}\cdot\frac{9}{s^2}} = \frac{9(b+16s)}{s^3+bs^2+9b+144s}$$

the characteristic equation is

$$s^3+bs^2+144s+9b=0$$

rearrange this we have

$$s^3+144s+b(s^2+9)=0.$$

or $$\frac{b(s^2+9)}{s^3+144s}+1=0$$

or $$\frac{b(s^2+9)}{s(s^2+144)}+1=0$$

①. find the poles and zeros

poles: $\pm 12j$, 0

zeros: $\pm 3j$.

② root loci on the real axis.

$$\frac{s^2+9}{s(s^2+144)}<0. \quad \therefore \quad s<0.$$

③ asymptotes of root loci

$n=3 \quad m=2, \quad \alpha=0.$

$$\angle(s-\alpha) = \angle s = \frac{180^\circ + l \cdot 360^\circ}{n-3} = 180^\circ(2l+1)$$

corresponds to the negative real axis.

(4) breakaway and break-in points.

$$\frac{6(s^2+9)}{s(s^2+144)} + 1 = 0$$

Solve for $[s(s^2+144)]' \cdot (s^2+9) - (s^2+9)'[s(s^2+144)] = 0$.

we have. $s^4 - \cancel{}\,117 s^2 + 144 \times 9 = 0$

the four roots are $\left\{ \begin{array}{l} \cancel{11.4506} \\ \cancel{-11.4506} \\ \cancel{3.1439} \\ -3.1439 \end{array} \right.$ $\left\{ \begin{array}{l} 10.228 \\ -10.228 \\ 3.5198 \\ -3.5198 \end{array} \right.$

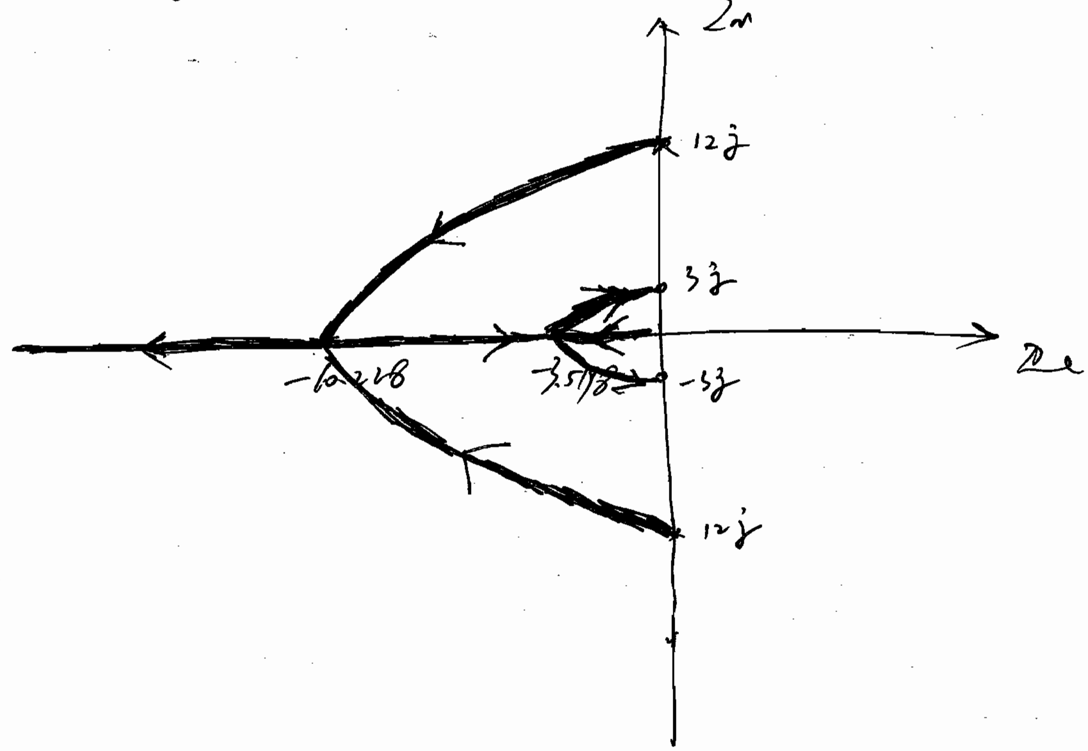
since only ~~-11.4506~~ -10.228 and ~~-3.1439~~ -3.5198 lie on the negative real axis. they are the only break in/away points.

(5). Departure/Arrival angle of complex poles (zeros)

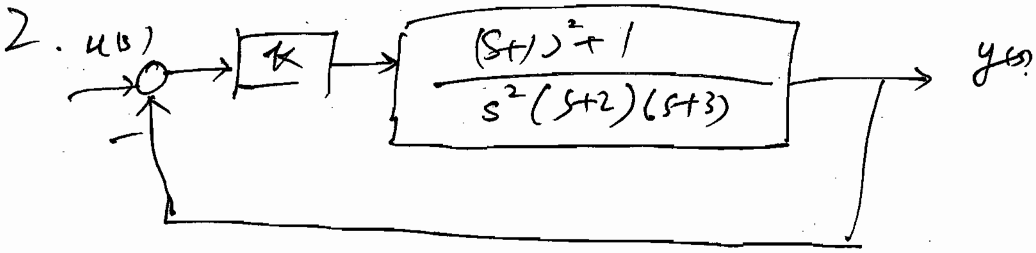
all poles and zeros lie on the imaginary axis, it can be easily determined that the departure/arrival angle of $\pm 12j$, $\pm 3j$ are 180°.

and zeros on imaginary axis.

⑤. From above, we can draw the root locus mannually.



2.



$$\frac{y(s)}{u(s)} = \frac{k \cdot \frac{(s+1)^2+1}{s^2(s+2)(s+3)}}{1 + k \cdot \frac{(s+1)^2+1}{s^2(s+2)(s+3)}}$$

the characteris equation is

$$k \cdot \frac{(s+1)^2+1}{s^2(s+2)(s+3)} + 1 = 0$$

① poles and zeros.

poles: $0, -2, -3.$ zeros: $-1 \pm j.$

② root loci on the real axis

since $\frac{(s+1)^2+1}{s^2(s+2)(s+3)} < 0.$

then $-2 < s < -3.$

③ asymptotes of root loci

$n=4, \quad m=2, \quad \alpha = \frac{0+(-2)+(-3)-(-2)}{2} = -1.5$

$$\angle(s-\alpha) = \frac{180^\circ + l \cdot 360^\circ}{n-m} = 90^\circ + l \cdot 180^\circ.$$

④. breakaway and break-in points.

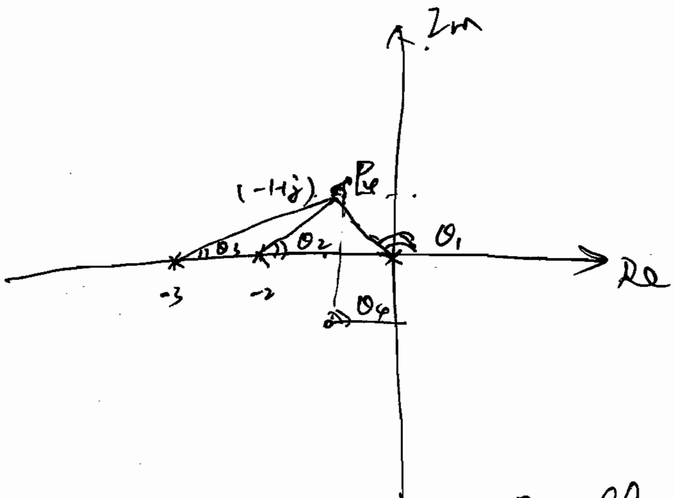
Solve for

$$[s^2(s+2)(s+3)]'[(s+1)^2+1] - [(s+1)^2+1]'[s^2(s+2)(s+3)] = 0$$

then $S = \begin{cases} 0 \\ -0.9593+1.8664j \\ -0.9593-1.8664j \\ -2.4848 \\ -1.0966 \end{cases}$ since only -2.4848 lies in $(-3,-2)$

it is the only breakaway point.

⑤. Departure / Arrive angle of complex zeros



$\theta_1 = 135°$

$\theta_2 = 45°$

$\theta_3 = 26.56°$

$\theta_4 = 90°$

$$\varphi + \theta_4 - (2\theta_1 + \theta_2 + \theta_3) = 180° + 360° \cdot l.$$

$$\therefore \quad \varphi = 71.56°$$

⑥. points where root loci may cross imaginary axis

when $s = j\omega$, $\omega \neq 0$

$$\frac{(s+1)^2+1}{s^2(s+2)(s+3)} = \frac{2-w^2+2jw}{-w^2(6-w^2+5jw)}$$

when it is real, we have

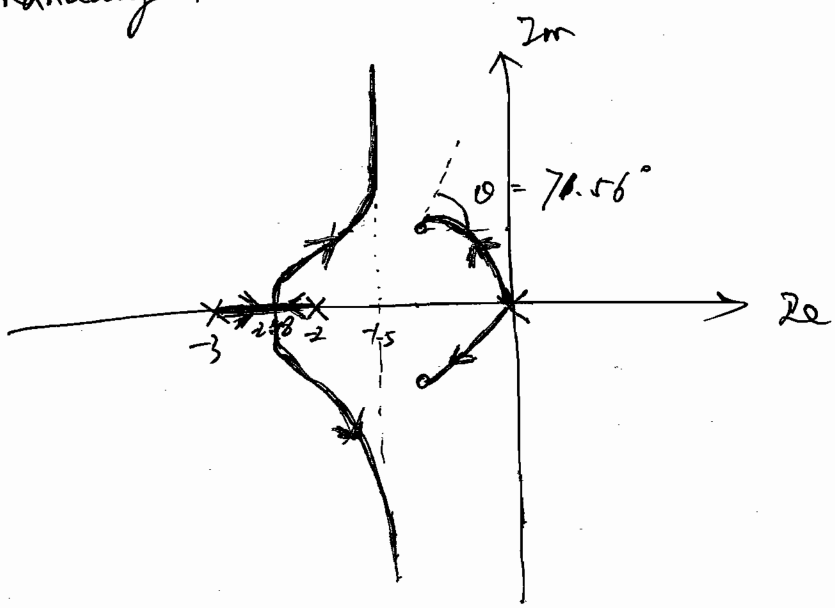
$$\frac{2-w^2}{6-w^2} = \frac{2w}{5w} = \frac{2}{5}$$

$$3w^2 = -2.$$

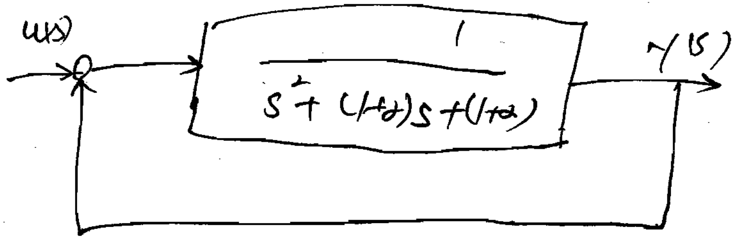
$\therefore$ there is no real w satisfying it.

$\therefore$ no point of root loci on imaginary axis

⑦ From above, we can draw the root loci manually.



3.



$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{s^2+(1+\alpha)s+(1+\alpha)}}{1+\frac{1}{s^2+(1+\alpha)s+(1+\alpha)}}$$

$$= \frac{1}{s^2+(1+\alpha)s+(2+\alpha)}$$

the characteristic equation is

$$s^2+(1+\alpha)s+(2+\alpha)=0$$

rearrange this we have

$$s^2+s+2+\alpha(s+1)=0$$

or $\frac{\alpha(s+1)}{s^2+s+2}+1=0$

①. poles and zeros.

poles: $-\frac{1}{2}\pm\frac{\sqrt{7}}{2}j$ zero: -1

② root loci on real axis

since $\frac{s+1}{s^2+s+2}<0$ then $s<-1$.

③ asymtotes of root loci;

$n=2 \quad m=1. \quad \alpha=0.$

$$\angle(s-\alpha) = \angle s = \frac{180^\circ + 360^\circ \cdot l}{n-m} = 180^\circ + 360^\circ \cdot l.$$

corresponds to the negative real axis.

④. breakaway and break-in points

solve for

$$(s^2+s+2)'(s+1) - (s+1)'(s^2+s+2) = 0$$

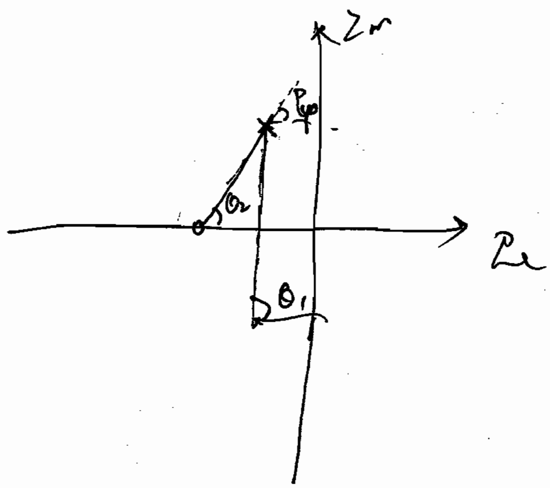
we have $s^2+2s-1=0$

$$\therefore \quad s = \begin{cases} -1+\sqrt{2} \\ -1-\sqrt{2} \end{cases}$$

since $-1-\sqrt{2} < -1$, it is the only

break-in point.

⑤ Departure / Arrival angle of complex poles



$\theta_1 = 90^\circ$

$\theta_2 = \operatorname{atan}^{-1}\sqrt{7} = 69.3^\circ$.

$\theta_2 - (\theta_1 + \varphi) = 180^\circ + l \cdot 360^\circ$

$\therefore \varphi = 159.3^\circ$

⑧. points where root loci may cross the imaginary axis

when $s = j\omega$, $\omega \neq 0$

$$\frac{s+1}{s^2+s+2} = \frac{1+j\omega}{2-\omega^2+j\omega}$$

when it is real. $\cancel{\frac{1}{2-\omega^2}} = 1.$

then $\frac{s+1}{s^2+s+2} = \frac{1+j\omega}{2-\omega^2+j\omega} = \frac{j\omega}{j\omega} = 1 > 0.$

$\cancel{\angle\left(\frac{s+1}{s^2+s+2}\right) = 0^\circ.}$ ~~not~~

$\angle = 0^\circ$. not on the loci;

$\therefore$ no intersection.

⑨. From above, we can now draw the root locus.