\[ \frac{y(s)}{w(s)} = \frac{\frac{b+16s}{s+5}}{1 + \frac{b+16s}{s+5} \cdot \frac{9}{s^2}} = \frac{9 (b+16s)}{s^3 + bs^2 + 96 + 144s} \]

The characteristic equation is

\[ s^3 + bs^2 + 144s + 96 = 0 \]

Rearrange this we have

\[ s^3 + 144s + b(s^2 + 9) = 0. \]

or

\[ \frac{b(s^2 + 9)}{s^3 + 144s} + 1 = 0 \]

or

\[ \frac{b(s^2 + 9)}{s(s^2 + 144)} + 1 = 0 \]

1. **Find the poles and zeros**
   - Poles: \( \pm 12 \), \( 0 \)
   - Zeros: \( \pm 3 \)

2. **Root Loci on the real axis**

\[ \frac{s^2 + 9}{s(s^2 + 144)} < 0 \]

\[ S < 0. \]

3. **Asymptotes of root loci**

\[ n = 3, \ m = 2, \ \alpha = 0. \]
\[ \angle (S-a) = \angle s = \frac{180^\circ + 1.36^\circ}{n-3} = 180^\circ (21+1) \]

corresponds to the negative real axis.

\( b \left( \frac{s^2+9}{s(s^2+16)} \right) + 1 = 0 \)

Solve for \[ s \left( s^2+16 \right) - (s^2+9) \left( s^2+16 \right) = 0 \]
we have \[ 5^0 - \frac{117}{5} + 1.44x9 = 0 \]

the four roots are

\[
\begin{array}{c|cc}
    & 77.4506 & 10.228 \\
    & 11.4506 & -10.228 \\
    & 2.1439 & 3.5198 \\
    & -3.1439 & -3.5198 \\
\end{array}
\]

since only \( -15.228 \) and \( -11.4506 \) lie on the negative real axis. they are the only break in frequency points.

\( 3. \) Departure/arrival angle of complex poles (zeros)

all poles and zeros lie on the imaginary axis, it can be easily determined that the departure/arrival angle of \( \pm 12.5^\circ, \pm 12.5^\circ \) are \( 180^\circ \).
5. From above, we can draw the root locus manually.
\[ \frac{y(t)}{u(t)} = k \cdot \frac{(s+1)^2 + 1}{s^2 (s+2)(s+3)} + \frac{1}{1 + k \cdot \frac{(s+1)^2 + 1}{s^2 (s+2)(s+3)}} \]

The characteristic equation is

\[ k \cdot \frac{(s+1)^2 + 1}{s^2 (s+2)(s+3)} + 1 = 0 \]

1. **Poles and Zeros.**
   - Poles: 0, -2, -3.
   - Zeros: -1 ± j.

2. **Root Loci on the Real Axis.**
   - Since \( \frac{(s+1)^2 + 1}{s^2 (s+2)(s+3)} \) is in \( \mathbb{C}_0 \),
   - then \(-2 < s < -3\).

3. **Asymptotes of Root Loci.**
   - \( n = 4, \ m = 2, \ d = \frac{0 + (2) + (3) - (2)}{2} = 1.5 \)
   - \( \angle (S + 2) = \frac{180^\circ + 1 \cdot 360^\circ}{n - m} = 90^\circ + 1 \cdot 180^\circ \).
5. Breakaway and break-in points.

Solve for

\[ \frac{s^2 (s+2) (s+3)}{(s+1)^2+1} \cdot \frac{1}{s^2 (s+2) (s+3)} = 0 \]

then

\[ S = \begin{cases} 0 & \text{since only } -2.4848 \text{ lies in } (-3, -2) \\ -0.9573 + 1.8664j \\ -0.9573 - 1.8664j \\ -2.4848 \\ -1.0366 \end{cases} \]

It is the only breakaway point.

6. Departure/Arrival angle of complex zeros:

\[ \theta_1 = 135^\circ \]
\[ \theta_2 = 45^\circ \]
\[ \theta_3 = 26.56^\circ \]
\[ \theta_4 = 9^\circ \]

\[ \phi + \phi_4 = (2\theta_1 + \theta_2 + \theta_3) = 180^\circ + 360^\circ - 1 \]

\[ \phi = 71.56^\circ \]

6. Points where root loci may cross imaginary axis:

when \( s = jw \), \( w = \)
\[
\frac{(s+1)^2+1}{s^2(s+2)(s+3)} = \frac{2-w^2 + 2i w}{-w^2 (6w^2 + 53w)}
\]

When it is real, we have

\[
\frac{2-w^2}{6-w^2} = \frac{2w}{5w} = \frac{2}{5}
\]

\[
3w^2 = -2,
\]

thus there is no real \( w \) satisfying it.

no point of root loci on imaginary axis.

\( \odot \) From above, we can draw the root loci manually.
\[ y(\infty) = \frac{1}{s^2 + (1+\alpha)s + (1+\alpha)} \]
\[ = \frac{1}{s^2 + (1-\alpha)s + (2+\alpha)} \]

The characteristic equation is

\[ s^2 + (1+\alpha)s + (2+\alpha) = 0 \]

Rearrange this we have

\[ s^2 + s + 2 + \alpha | s + 1 | = 0 \]

or \[ \frac{2(s+1)}{s^2 + s + 2} + 1 = 0 \]

0. Poles and zeros
   - Poles: \( -\frac{1}{2} \pm \frac{\sqrt{3}}{2} j \)
   - Zero: \(-1\)

0. Root loci on real axis
   - Since \( \frac{s+1}{s^2 + s + 2} < 0 \) when \( S < -1 \)
3) asymptotes of root loci:

\[ n = 2 \quad m = 1 \quad \Delta = 0. \]

\[ \angle (s^2) = \angle s = \frac{180^\circ + 360^\circ \cdot 1}{n-m} = 180^\circ + 360^\circ \cdot 1. \]

corresponds to the negative real axis.

4) Breakaway and break-in points.

Solve for

\[ (s^2 + s + 2)(s + 1) - (s + 1)(s^2 + s + 2) = 0 \]

we have

\[ s^2 + s - 1 = 0 \]

\[ s = \frac{-1 \pm \sqrt{5}}{2} \]

Since \(-1 < -\Delta < -1\), it is the only

break-in point.

5) Departure/arrival angle of complex poles

\[ \theta_1 = 90^\circ \]

\[ \theta_2 = \arctan \frac{1}{1} = 45^\circ \]

\[ \theta_2 - (\theta_1 + \Psi) = 180^\circ + 1.36^\circ \]

\[ \Psi = 159.3^\circ \]
6. Points where root loci may cross the imaginary axis

When \( s = jw \), \( w \neq -1 \):

\[
\frac{s+1}{s^2+5s+2} = \frac{1+3w}{2-w^2+jw}
\]

When it is real, \( \frac{s}{2-\omega} = 1 \):

\[
\frac{s+1}{s^2+5s+2} = \frac{1+3w}{2-w^2+jw} = \frac{jw}{jw} = 1 \to 0.
\]

\[\frac{\phi}{2\pi} = 0^\circ \text{ root on the loci; no intersection.}\]

7. From above, we can now draw the root locus.