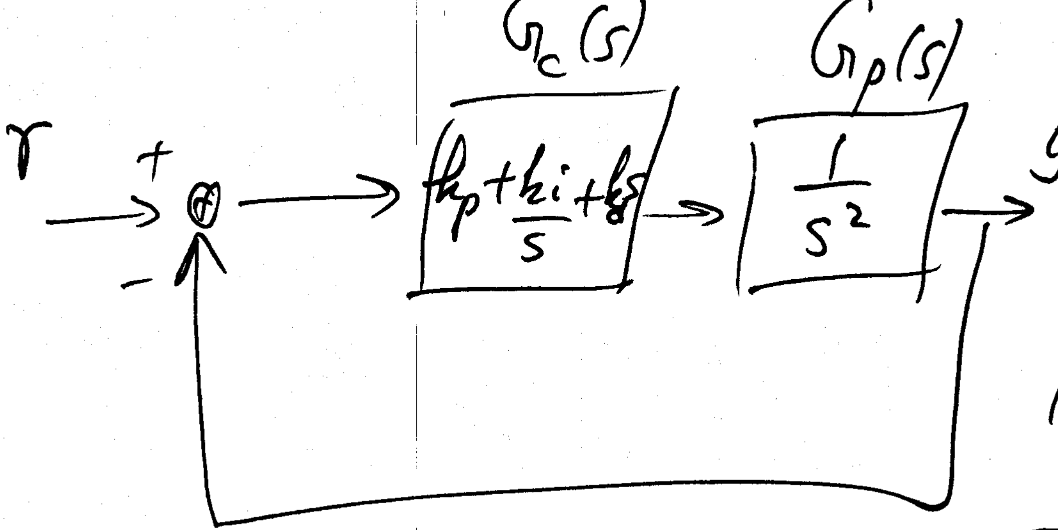


closed loop transfer function $\dfrac{\overbrace{G_c G_p}^{\text{loop gain}}}{\underbrace{1 + G_c G_p}_{\text{return difference}}}$

Closed loop poles: zeros of $\left| 1 + G_c(s) G_p(s) \right|$

Root locus: Determine the location of the closed loop poles as a function of a single gain (parameter) in $G_c(s) G_p(s)$.

Nyquist: Determine the stability of the closed loop system based on the Nyquist plot of $G_c(s) G_p(s)$.

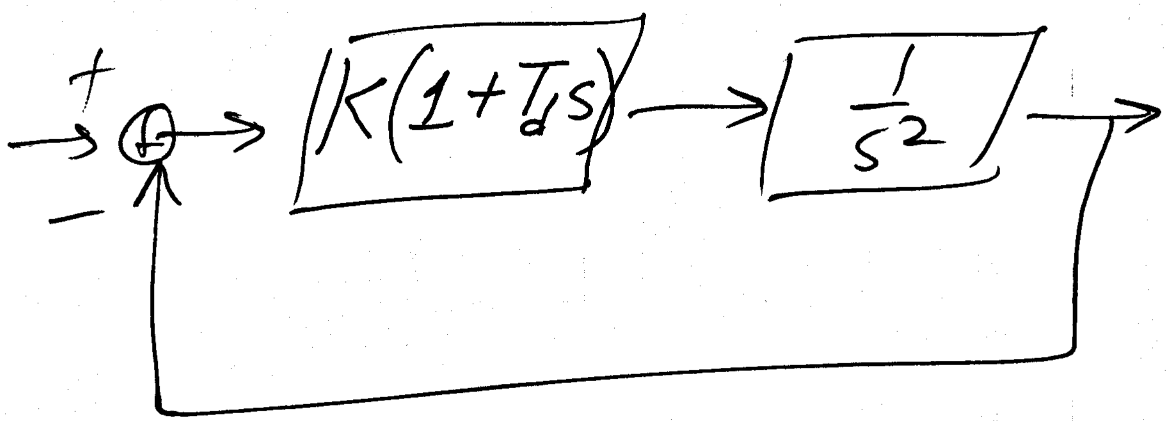
$Re(G_c(j\omega) G_p(j\omega))$ vs. $Im(G_c(j\omega) G_p(j\omega))$

Determine the zeros of

Root Locus: $\boxed{1 + G_c(s)\,G_p(s)}$

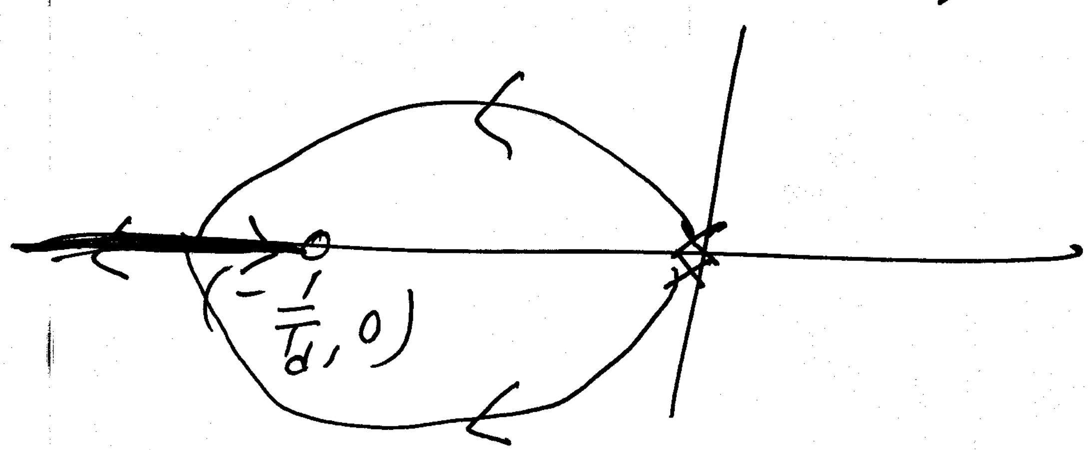
$\Updownarrow$

$$1 + \underline{k}\,F(s)$$



loop gain: $K\frac{(1+T_d s)}{s^2}$

Return difference: $1 + K\boxed{\frac{(1+T_d s)}{s^2}}$ given



what if $1 + G_c G_p$ cannot be written as $1 + k\bar{F}(s)$?

k — known

loop gain

$$\frac{\left(k_p + \frac{k_i}{s} + k_d s\right)}{s^2}$$

chosen

return difference

$$1 + \left(\frac{k_p + k_d s + \frac{k_i}{s}}{s^2}\right)$$

root locus parameter

$$= \underbrace{1 + \frac{k_p + k_d s}{s^2}}_{\text{known}} + \frac{k_i}{s^3}$$

known

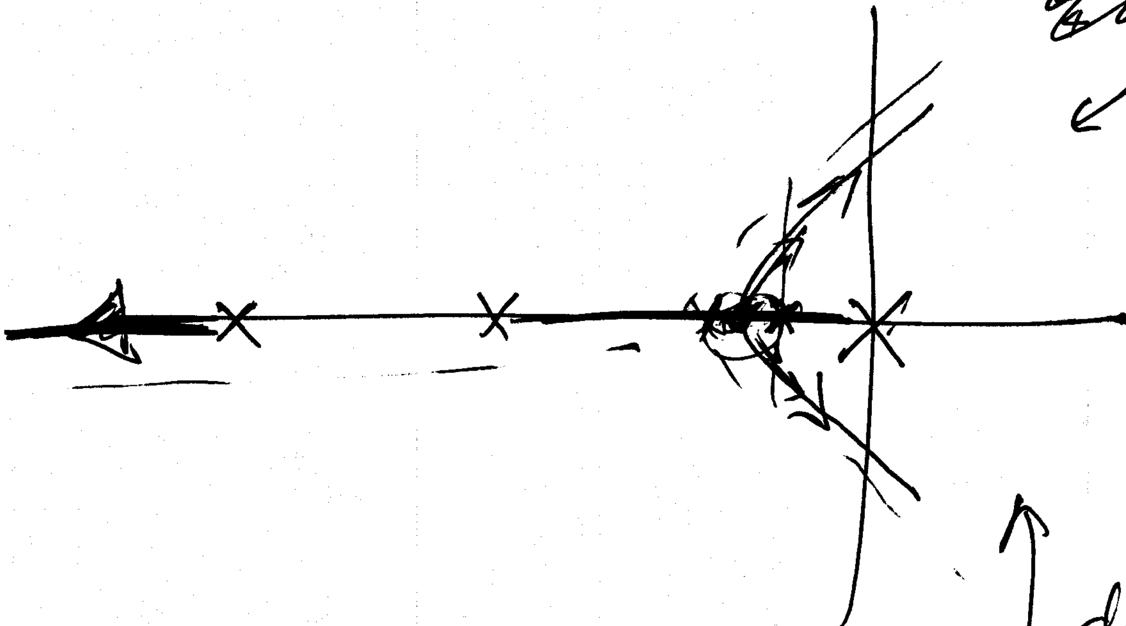
$$= \left(1 + \frac{k_p + k_d s}{s^2}\right)\left(1 + k_i \frac{s^2}{s^2 + k_d s + k_p} \cdot \frac{1}{s^3}\right)$$

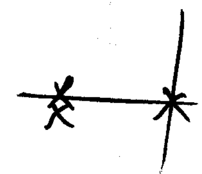
$$\frac{s^2 + k_d s + k_p}{s^2} \qquad \text{“}\frac{1}{s}\text{”}$$

$$= \left(\frac{s^2 + k_d s + k_p}{s^2}\right) \boxed{\left(1 + k_i \frac{1}{s(s^2 + k_d s + k_p)}\right)}$$

~~$\frac{k_d}{4} - k_p$~~

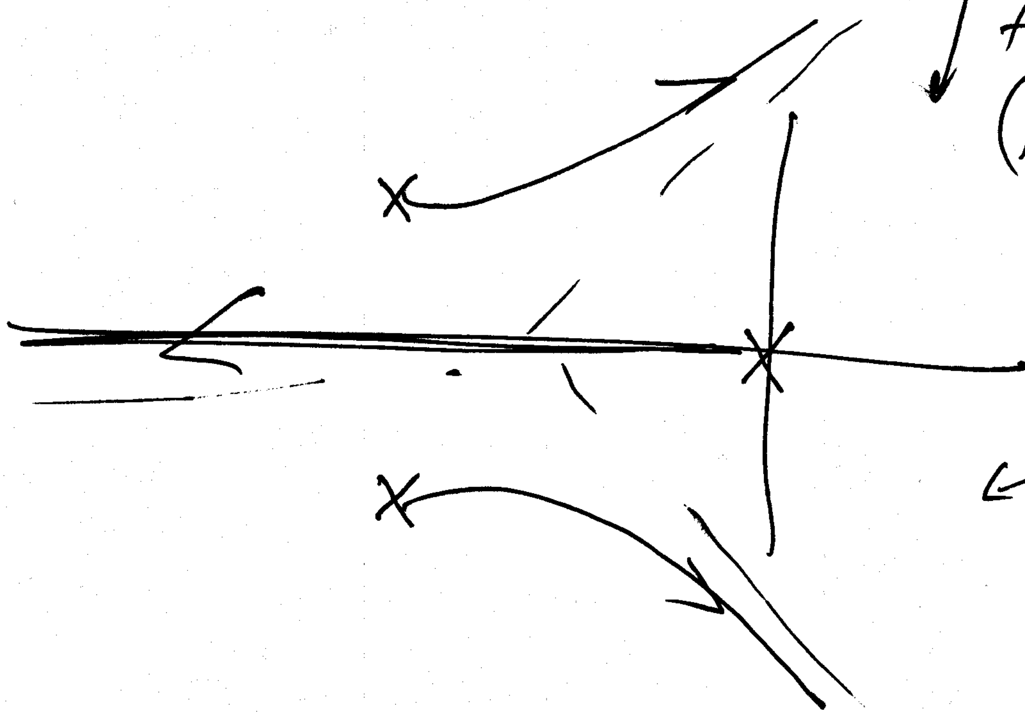
$k_d \geq 4 k_p$



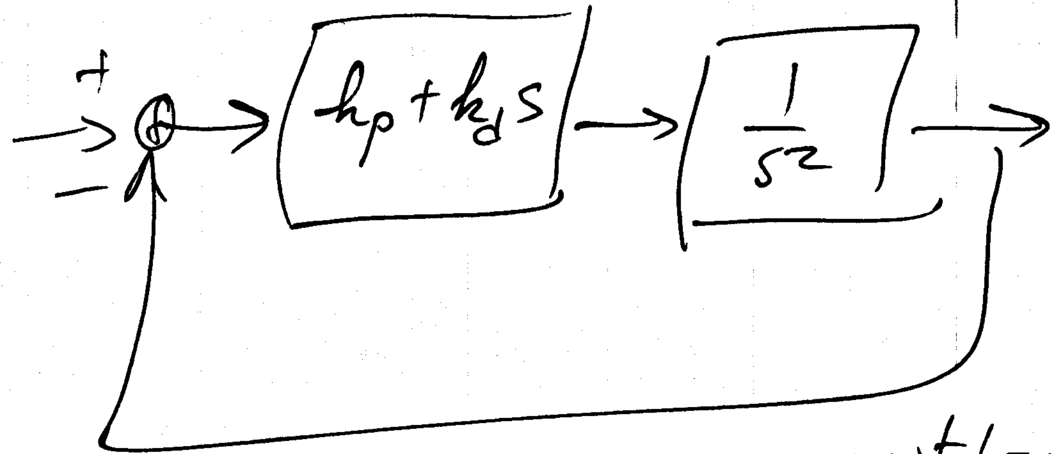


$k_d = 4 k_p$

depending on the values of (K_p, K_d)



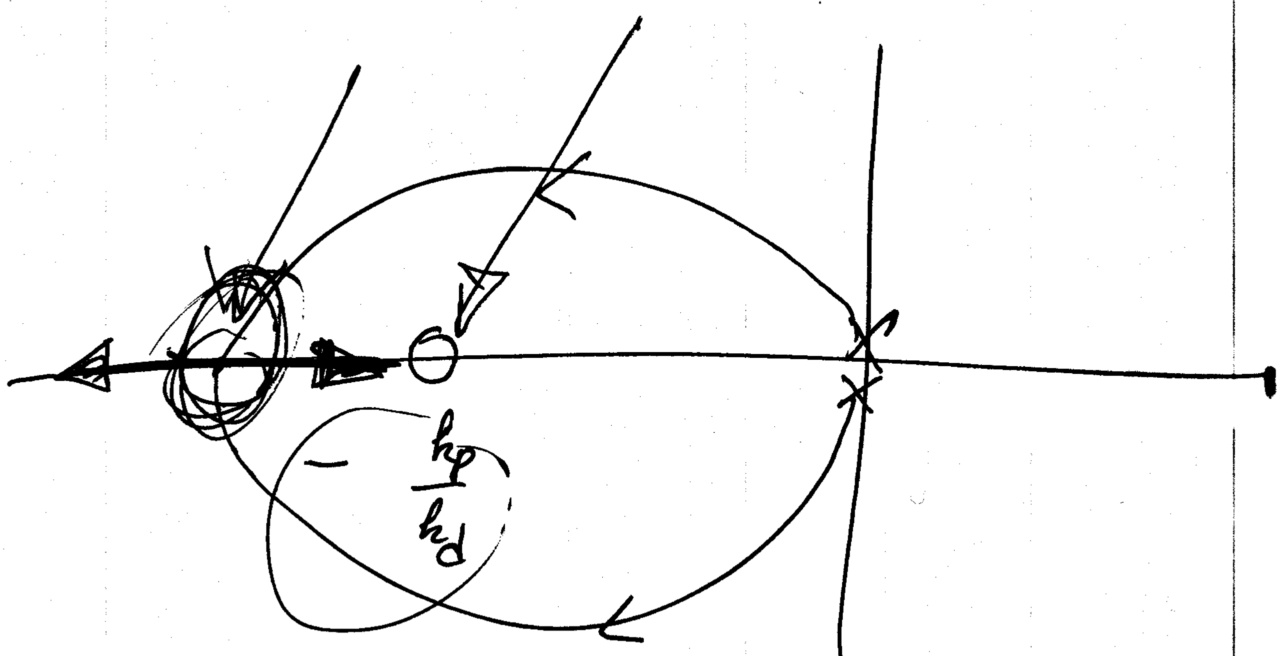
$k_d < 4 k_p$



root locus parameter

$$\frac{k_p + k_d s}{s^2} = \frac{k_p\left(1 + \boxed{\frac{k_d}{k_p}}\, s\right)}{s^2}$$

to be chosen



$$1+\left(\frac{b+16s}{b+s}\right)\frac{9}{s^2}$$

$$=\frac{(b+s)s^2+(b+16s)9}{(b+s)\,s^2}$$

$$s^3+s^2b+9b+144s$$

$$=\underbrace{s^3+144s}_{\text{known}}+b\underline{(9+s^2)}$$

$F(s)$

$$=(s^3+144s)\left(1+b\left(\frac{9+s^2}{s^3+144s}\right)\right)$$

$$1+\frac{1}{s^2+(1+\alpha)s+(1+\alpha)}$$

$$M(q_d) = \begin{bmatrix} a_1 + 2a_2\cos\theta_{2d} & a_3 + a_2\cos\theta_d \\ a_3 + a_2\cos\theta_d & a_4 \end{bmatrix}$$

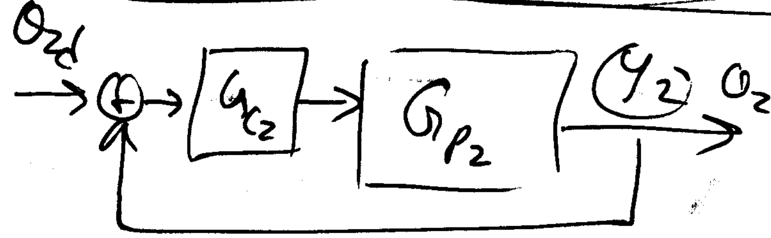
$$M(q_d)\begin{bmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix} = \begin{bmatrix} (a_1 + 2a_2\cos\theta_{2d})\ddot\theta_1 + \cancel{(a_3 + a_2\cos\theta_d)\ddot\theta_2} \\ (a_3 + \cancel{a_2\cos\theta_d})\ddot\theta_1 + a_4\ddot\theta_2 \end{bmatrix}$$

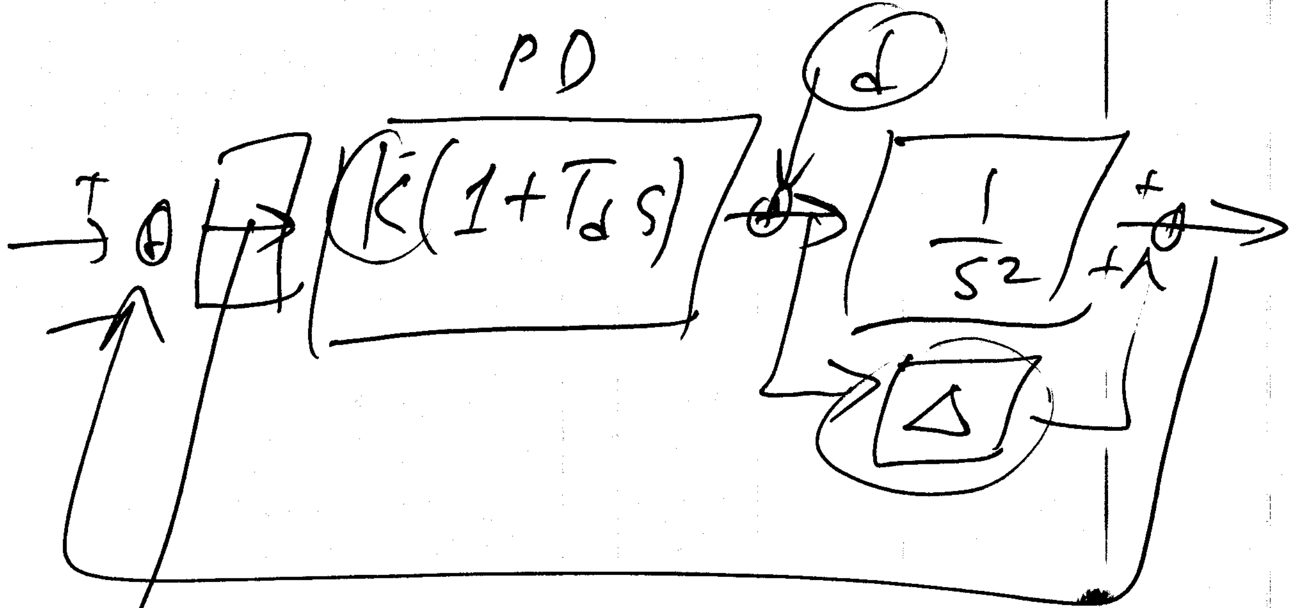
$$D\dot q = \begin{bmatrix} F_{v_1}\dot\theta_1 \\ F_{v_2}\dot\theta_2 \end{bmatrix} \qquad u = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}$$

$$y_1 = \theta_1 \qquad y_1 = \frac{\tau_1}{(a_1 + 2a_2\cos\theta_{2d})s^2 + F_{v_1}s}$$

$$y_2 = \theta_2$$

$$\boxed{y_2 = \frac{\tau_2}{a_4 s^2 + F_{v_2} s}}$$





α ← gain

$e^{j\phi}$ ← phase

$\boxed{\frac{(1+T_d s)}{s^2}}$ → Bode plot

$\boxed{\frac{1+T_d s}{s^2}}$ → Nyquist

→ Bode

gain/phase margin

For Next Tuesday

Equilibrium and Linearization

Equilibrium (for zero input) : Any constant q.

Linearize about q_d: $\underline{M(q_d)}\ddot{q} + \underline{\underline{D}}\dot{q} = u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

replace $\cos\theta_2$ by $\cos\theta_{2d}$ — viscous friction

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = y = q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

$$\boxed{\begin{matrix} u_1 \longrightarrow y_1 \\ u_2 \longrightarrow y_2 \end{matrix} \; ?}$$