Today (11/18/05)

- Today
  - State space description of LTI systems
  - Put the 2-link system in project in SS form
  - Eigenvalues
  - Homework related to project

Project

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N(\dot{q}) = u$$

Equilibrium (for zero input) : Any constant *q*.

Linearize about q_d : $$M(q_d)\ddot{q} + D\dot{q} = u$$

Eigenvalues and Eigenvectors

$$A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$$

To find the eigenvalues and eigenvectors of A solve for σ and e from $Ae = \sigma e$

$$Ae = \sigma e \Leftrightarrow (\sigma I_{2\times 2} - A)e = 0 \Leftrightarrow \det(\sigma I - A) = 0$$

$$\sigma I - A = \begin{bmatrix} s+2 & -1 \\ -1 & s+2 \end{bmatrix} \quad \det(\sigma I - A) = s^2 + 4s + 3 = (s+3)(s+1)$$

eigenvalues of $A: \{-1, -3\}$

eigenvectors of $A:$

$$\sigma_1 = -1 : (\sigma_1 I - A)e_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} e_1 \Rightarrow e_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\sigma_2 = -3 : (\sigma_2 I - A)e_2 = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} e_2 \Rightarrow e_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

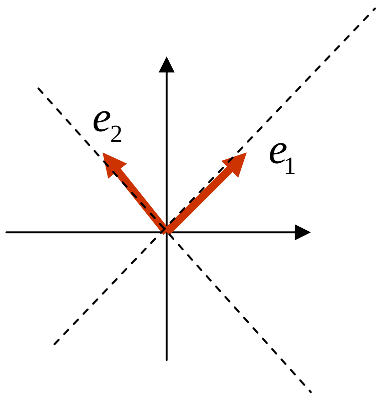
Eigenvalues and Eigenvectors

$$Ae_1 = \sigma_1 e_1 \ , Ae_2 = \sigma_2 e_2$$

$$A\begin{bmatrix} e_1 & e_2 \end{bmatrix} = \begin{bmatrix} \sigma_1 e_1 & \sigma_2 e_2 \end{bmatrix} = \begin{bmatrix} e_1 & e_2 \end{bmatrix}\begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

$$\begin{bmatrix} e_1 & e_2 \end{bmatrix}^{-1} A\begin{bmatrix} e_1 & e_2 \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$$

Check: $\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix}$



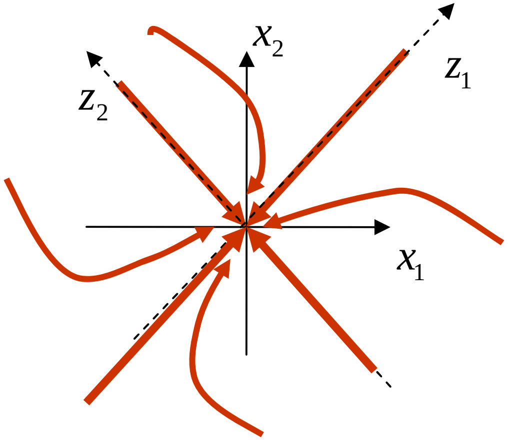
Eigenvalues and Eigenvectors

Now consider $\dot{x} = Ax.$ Let $M = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

$$M^{-1}\dot{x} = \left(M^{-1}AM\right)M^{-1}x.$$

Define $z = M^{-1}x.$

Then $\dot{z} = \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} z$ or $z(t) = \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-3t} \end{bmatrix} z(0)$ or $x(t) = M^{-1}\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-3t} \end{bmatrix} Mx(0)$



Discrete Time Systems

Under zero-order-hold, equivalent discrete time system is

$$x_{k+1} = A_d\, x_k + B_d\, u_k$$

Full state feedback control:

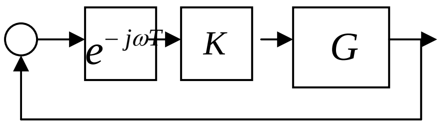
$$u_k = -F\,(x_k - x_d)$$

Pole placement can again be used to place the closed loop poles ***eig(**A_d **-** $B_d F$**)*** **within the unit circle.**

Homework (due 11/29/05)

- Put the 2-link system in the project in state space form.
- Linearize the system. Find poles of the system.
- Obtain $\tau_1 \rightarrow \theta_1$ and $\tau_2 \rightarrow \theta_2$ transfer functions.
- Consider the following the design spec: closed loop stability, rise time about ½ sec, 2% settling time, less than 5% overshoot, no more than 1 sec, 6dB gain margin (gain scaling from ½ to 2) and 10ms delay tolerance.
  - Use root locus to design a controller to achieve the design spec.
  - Assume full state is available (e.g., by using the washout filter). Use state space pole place design method to achieve the design spec.
- Simulate the linear and nonlinear system responses using the above controllers.

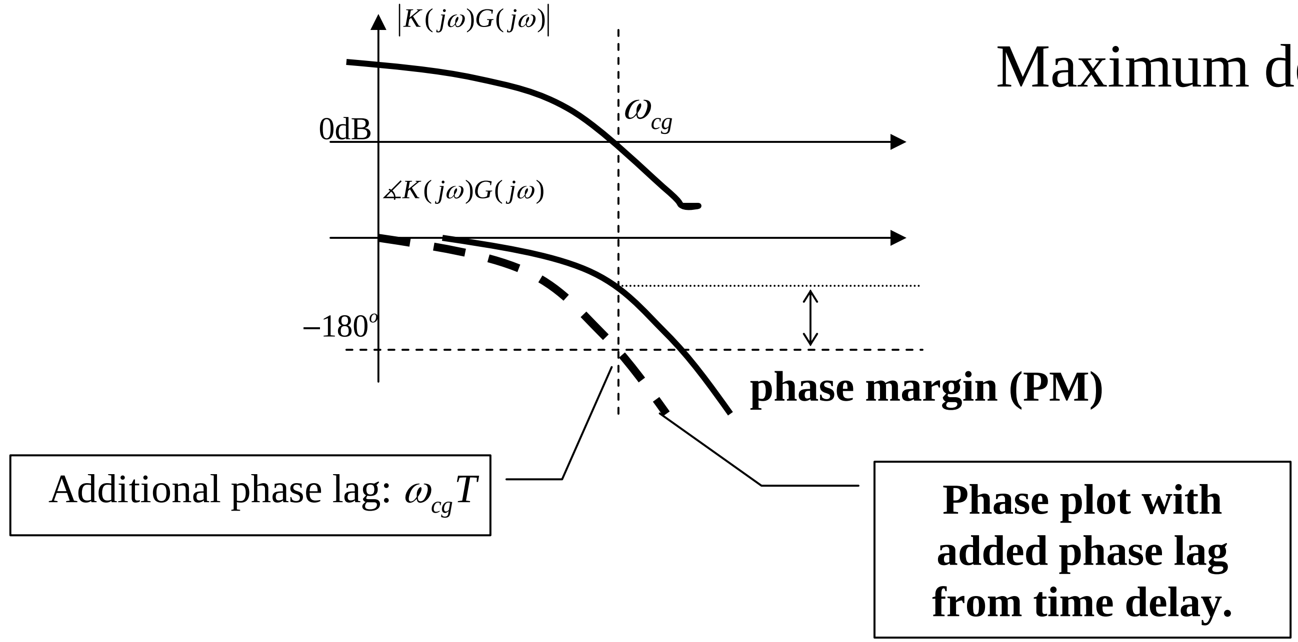
Time Delay



Time delay adds a phase shift of -ωT.

Boundary of stability: PM=$\omega_{cg}T$

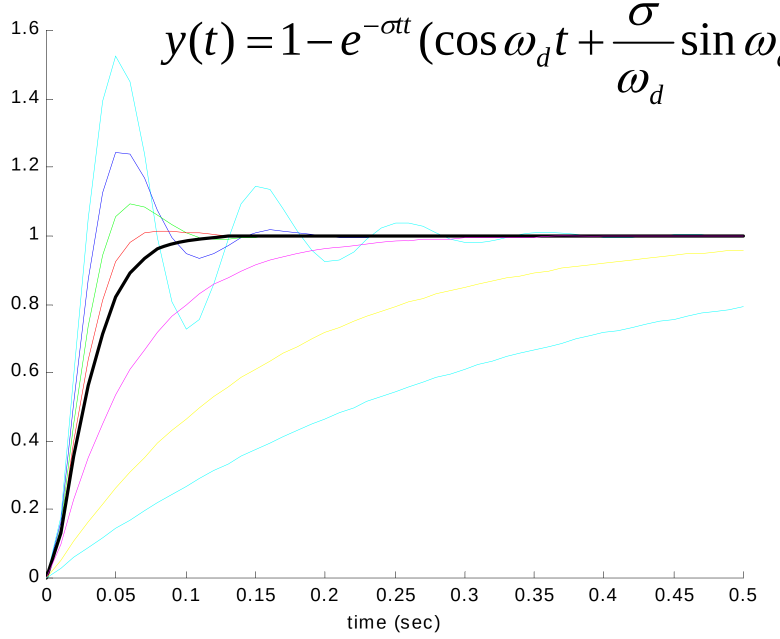
Maximum delay: $T_{\max} = \dfrac{\text{PM}}{\omega_{cg}}$



Time Domain Spec vs. Poles

Use 2^{nd} order system response to generate rule of thumb

$$y(t) = 1 - e^{-\sigma t t}(\cos\omega_d t + \frac{\sigma}{\omega_d}\sin\omega_d t)$$



- **Rise time:choose ζ=.5 as an average**

$$t_r \cong \frac{1.8}{\omega_n}$$

- **Peak time: set and solve for t_p and M_p**

$$t_p \cong \frac{\pi}{\omega_d} \qquad M_p \cong e^{-\pi\zeta/\sqrt{1-\zeta^2}}$$

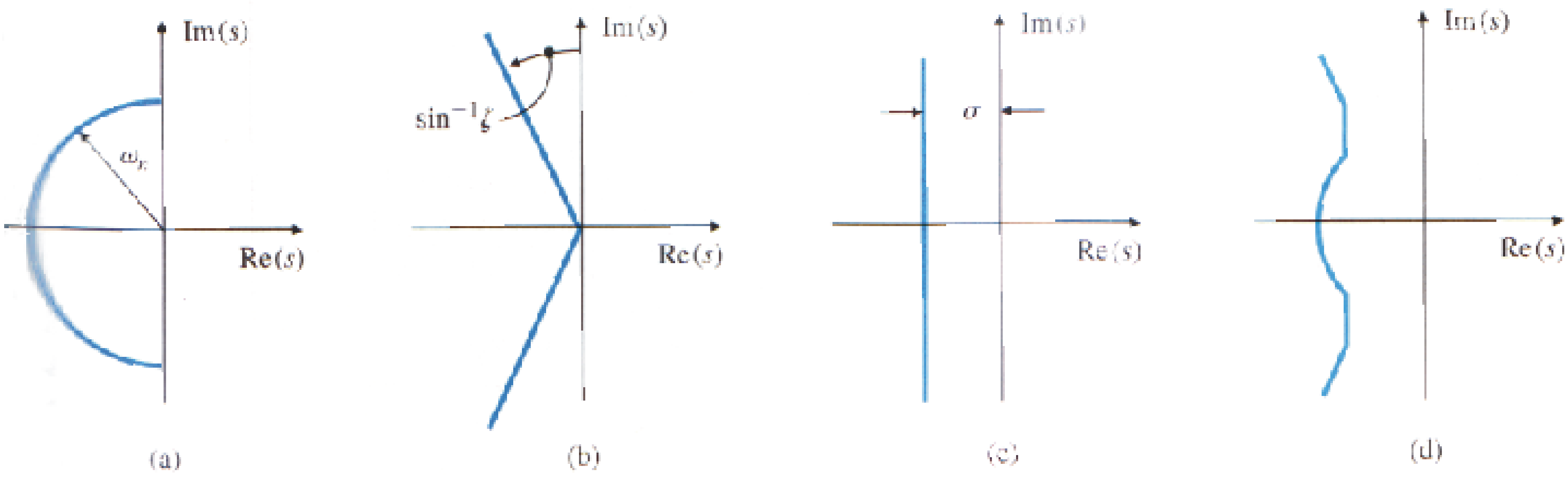
- **1% settling time:**

$$t_s \cong \frac{4.6}{\sigma}$$

Rule of Thumb for Pole Locations

Given time domain specifications: t_r , M_p , t_s , choose target pole locations as:

$$\omega_n \geq \frac{1.8}{t_r} \qquad \zeta \geq \zeta(M_p) \qquad \sigma \geq \frac{4.6}{t_s}$$



Project

Project spec:

$$t_r < .5\text{sec},\ t_s < 1\text{sec},\ M_p < 0.05$$

Rule of thumb:

$$\omega_n \geq \frac{1.8}{t_r} = 3.6 \text{ rad/sec}$$

$$\zeta \geq \zeta(M_p) \quad \zeta / \sqrt{1-\zeta^2} = \ln M_p / (-\pi) = b$$

$$\zeta = \sqrt{\frac{b^2}{1+b^2}} = .7 \quad (70\%)$$

$$\sigma = \zeta\omega \geq \frac{4.6}{t_s} = 4.6\text{rad/sec}$$