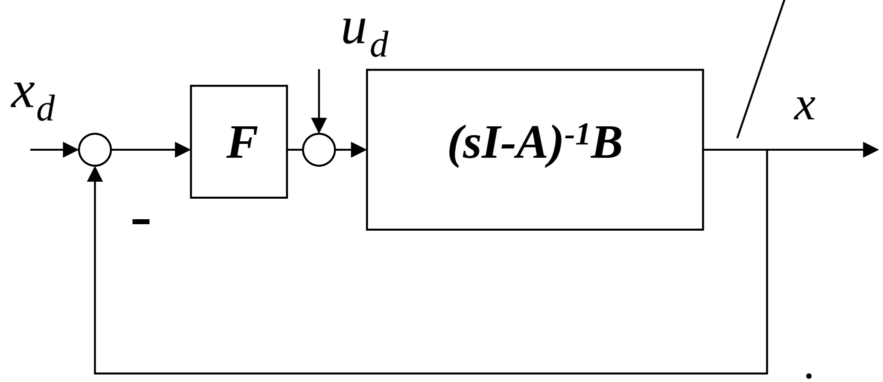
Today (11/22/05)

- Today
  - Eigenvalues and eigenvectors
  - Full state feedback
  - Tracking control
  - Discretization
  - Observer

Tracking Control

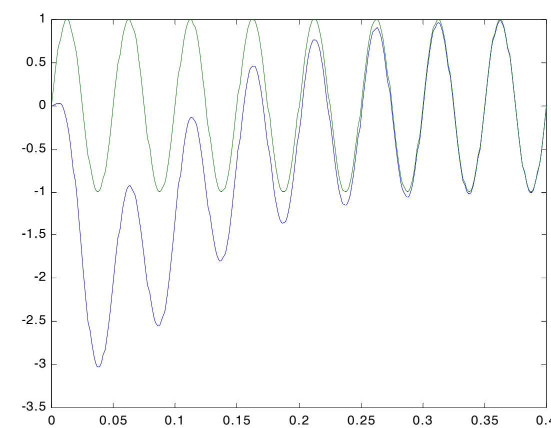


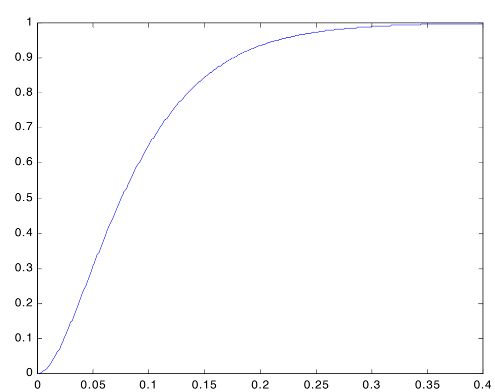
Gain and phase margins can be used to assess robustness wrt loop gain variation

$$u = -F(x - x_d) + u_d$$
$$= -Fx + (Fx_d + u_d)$$

$$\dot{x}_d = Ax_d + Bu_d$$
$$y_d = Cx_d + Du_d$$

- Tracking: Given y_d, find (x_d, u_d) such that
- Set Point: $\begin{bmatrix} x_d \\ u_d \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ y_d \end{bmatrix}$





Tracking Control

Example: $G(s) = 1/(s^2 + 2.427s + 1)$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2.427 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Set point control:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_d \\ u_d \end{bmatrix} = \begin{bmatrix} 0 \\ y_d \end{bmatrix} \Leftrightarrow \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2.427 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{d_1} \\ x_{d_2} \\ u_d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ y_d \end{bmatrix}$$

$$x_{d_1} = y_d,\ x_{d_2} = 0,\ u_d = y_d$$

Tracking control:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_d \\ u_d \end{bmatrix} = \begin{bmatrix} \dot{x}_d \\ y_d \end{bmatrix} \Leftrightarrow \begin{bmatrix} 0 & 1 & 0 \\ -1 & -2.427 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{d_1} \\ x_{d_2} \\ u_d \end{bmatrix} = \begin{bmatrix} \dot{x}_{d1} \\ \dot{x}_{d2} \\ y_d \end{bmatrix}$$

$$y_d = A \sin \omega t$$

$$x_{d1} = A \sin \omega t$$

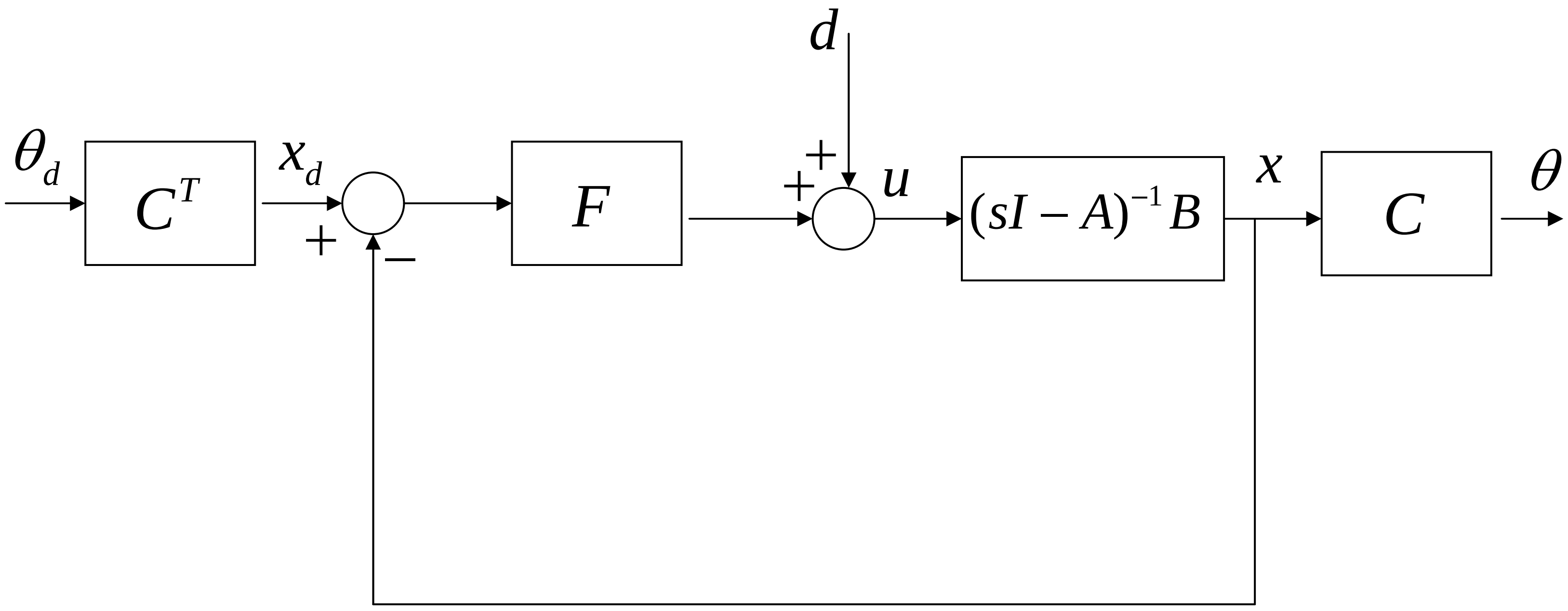
$$x_{d2} = \dot{x}_{d1} = A\omega \cos \omega t$$

$$u_d = x_{d1} + 2.427 x_{d2} + \dot{x}_{d2}$$

$$= A \sin \omega t + 2.427 A\omega \cos \omega t - A\omega^2 \sin \omega t$$

Project

Full state feedback:



What if the state is not available? (In Project 3, only θ can be measured.)

State Observer based Control

Idea: Estimate the state and use it in the full state feedback control as if it is the actual state.

- **How do we build a state estimator?**
- **Will the overall system be stable?**
- **How would the performance and robustness of the overall system change?**

Full State Observer

Idea:

- **Build a replica of the system**

Plant: $\dot{x} = Ax + Bu \quad ; \quad x(0) = x_0$

Replica: $\dot{\hat{x}} = A\hat{x} + Bu \quad ; \quad \hat{x}(0) = ?$

- **Use the output estimation error to reduce the state error due to initial condition mismatch**

True output: $y = Cx + Du$

Estimated output: $\hat{y} = C\hat{x} + Du$

Output estimation error: $\hat{y} - y = C\hat{x} + Du - y$

Full State Observer

$$\dot{\hat{x}} = \underbrace{A\hat{x} + Bu}_{\text{replica of plant}} \underbrace{- L(\underbrace{\underbrace{C\hat{x} + Du}_{\text{estimated output}} - y}_{\text{output estimation error}})}_{\text{correction based on output estimation error}} \qquad \hat{x}(0) = 0$$

Will It Work?

Check the stability of $x - \hat{x}$:

$$\dot{x} - \dot{\hat{x}} = \left(Ax + Bu\right) - \left(A\hat{x} + Bu\right) + L(C\hat{x} + Du - y); \quad x(0) - \hat{x}(0) = x_0$$
$$= A(x - \hat{x}) + LC(\hat{x} - x)$$
$$= (A - LC)(x - \hat{x})$$

If *eig(A-LC)* are all stable (in left half plane), then $\hat{x}$ will converge to *x* for what ever initial condition.

How do we choose L?

Fact: $eig(A)=eig(A^T)$

Therefore, $eig(A-LC) = eig(A^T-C^TL^T)$

This is exactly the same as the full state feedback case except (A,B) **is replaced by** (A^T,C^T)**.**

Rule of Thumb: Observer poles should be 5-10 times faster than the controller poles.

What about the closed loop?

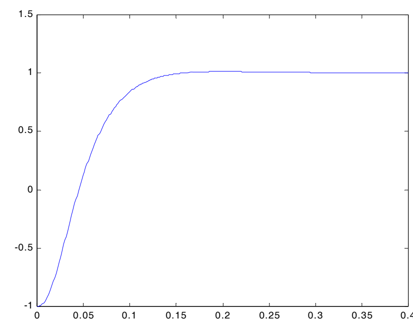
Separation Theorem:

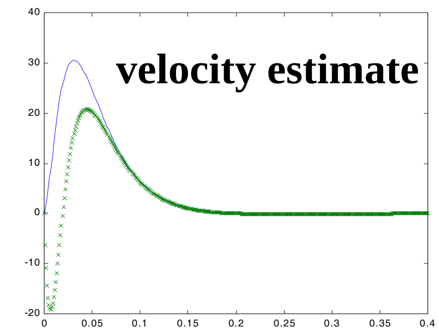
Replace x in full state feedback by state estimate

$$u = -F\hat{x}$$

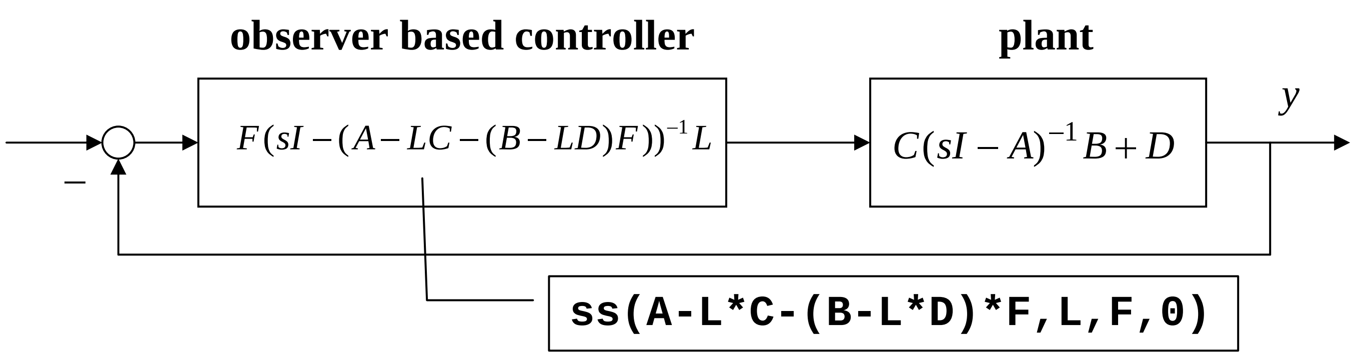
$$\dot{\hat{x}} = A\hat{x} + Bu - L(C\hat{x} + Du - y)$$

$$= (A - LC - (B - LD)F)\hat{x} + Ly$$





Closed loop poles given by full state feedback poles eig(A-BF) and observer poles eig(A-LC)



Exercise

- **Consider**

$$G(s) = 1/(s^2 + 2.427s + 1)$$

- **Obtain a state space representation**
- **Design a full state feedback controller. Tune it for "good" step response.**
- **Design an observer and combine with the same full state feedback controller. Tune the observer for good step response.**