1st order vector diff. eq'n

$$\boxed{\begin{aligned} \dot{x} &= \underbrace{f(x,u)} \\ y &= h(x,u) \end{aligned}}$$

state evolution equation

output equation (static)

$$\left\{\begin{aligned} & M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + N(\dot{\theta}) = \tau \\ & y = \theta \end{aligned}\right.$$

2 inputs and 2 outputs

state space representation

Define ← non-unique

$$x_1 = \theta - \theta_d$$
$$x_2 = \dot{\theta}$$

Legendre Transformation

(or $x_1 = \theta - \theta_d$

$\underbrace{x_2 = M(\theta_d)}\,\dot{\theta}$)

generalized momentum

$$\left\{\begin{aligned} \dot{x}_1 &= \dot{\theta} = x_2 \\ \dot{x}_2 &= \ddot{\theta} = M^{-1}(\theta)\left(-\left(C(\theta,\dot{\theta})\dot{\theta} + N(\dot{\theta})\right) + \tau\right) \\ &= M^{-1}(x_1)\left(-\left(C(x_1,x_2)x_2 + N(x_2)\right) + \tau\right) \end{aligned}\right.$$

Linearize (expand RHS as a Taylor series & then keep the linear term only)

$$\dot{x}_1 = x_2$$

$$\underbrace{M(x_1)}\dot{x}_2 = \underbrace{-\cancel{C(x_1,x_2)x_2}} - N(x_2) + \tau$$

$$\left(\underbrace{M(x_{1_d})}_{\text{constant matrix}} + \underbrace{\text{higher order terms}}\right)\dot{x}_2 \qquad N(x_2) \to \underbrace{\begin{bmatrix} F_{v_1} & 0 \\ 0 & F_{v_2} \end{bmatrix}}_{D}\underbrace{\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}}_{x_2}$$

$x_{1_d} = \theta_d$

$\dot{x}_2$: linear term

$\therefore$ Linearization of $M(x_1)\dot{x}_2 = \underbrace{M(\theta_d)}_{\text{const. matrix}}\dot{x}_2$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -M^{-1}(\theta_d)D\,x_2 + M^{-1}(\theta_d)\tau$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I \\ 0 & -M^{-1}(\theta_d)D \end{bmatrix}}_{A}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ M^{-1}(\theta_d) \end{bmatrix}}_{B}\tau$$

$$Y = \underbrace{\begin{bmatrix} I & 0 \end{bmatrix}}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\dot{x} = Ax + Bu + d$$

$$y = Cx$$

$$\dot{q} = y$$

$$\begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}}_{\underline{\underline{\bar{A}}}} \begin{bmatrix} x \\ q \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\underline{\underline{\bar{B}}}} u + \begin{bmatrix} d \\ 0 \end{bmatrix}$$

const.

$$u = -\bar{F} \begin{bmatrix} x \\ q \end{bmatrix}$$

$\mathrm{eig}(\bar{A} - \bar{B}\bar{F})$ are all stable, then

$$\begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} \to 0 \implies \dot{q} \to 0 \implies y \to 0$$

$$\begin{bmatrix} y \\ q \end{bmatrix} = \underbrace{\begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}}_{\bar{C}} \begin{bmatrix} x \\ q \end{bmatrix}$$

Design an observer using $(\bar{A}, \bar{B}, \bar{C})$

find

$\bar{L} \ni \text{eig}(\bar{A} - \bar{L}\bar{C})$ are stable.

Project due on 12/12

instead on 12/9 !!

Homework (due 11/29/05)

- Put the 2-link system in the project in state space form.
- Linearize the system. Find poles of the system.
- Obtain $\tau_1 \rightarrow \theta_1$ and $\tau_2 \rightarrow \theta_2$ transfer functions.
- Consider the following the design spec: closed loop stability, rise time about ½ sec, 2% settling time, less than 1 sec, no more than 5% overshoot, 6dB gain margin (gain scaling from ~~½ to 2) and 10ms~~ delay tolerance. NL
  - Use root locus to design a controller to achieve the design spec.
  - Assume full state is available (e.g., by using the washout filter). Use state space pole place design method to achieve the design spec. for linearized system
- Simulate the linear and nonlinear system responses using the above controllers.

For Next Tue { • Incorporate observer in your S.S. design 1ms + discretization
• Tune controller for the NL system performance