ECSE 4460  Control Systems Design

Progress Report, Progress Presentation, State Space Method

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http://www.cat.rpi.edu/~wen/ECSE446S06/
Lab Sign Up

- Leave real-time system ready for next group.
- Sign up for 2-hr slot/day and email to entire class.
- Wait at least one hr before signing up for more slots.
- Send email out to cancel your time slots also.

- Final presentation postponed until 5/3 (Wed 6-9).
  Meeting on 4/27 cancelled.
From Project Proposal (Reminder)

- Capitalize Coulomb.
- Don’t use “I” or “You”.
- Use punctuation marks in equations.
- Refer to Appendices in the report and explain the relevance.
- You should use simulation to justify your component selection.
- Use block diagrams to show control architecture.
- PID is not the only choice for control design!
- Number the bibliography and refer to them in report.
- Never regurgitate stuffs from textbook – show how a particular technique is relevant and applicable.
Progress Report

- This is a pre-cursor to your final report. Good proposal and progress report will save you a lot of time for the preparation of final report.

- Organization:
  - Preliminary pages: title page, executive summary, table of content, list of figures, list of tables
  - Body: introduction, preliminary results (simulation, model validation, experimental results, comparison with specification), summary of progress, bibliography (IEEE style)
  - Appendix
  - Statement of contribution
Project Progress Presentation

- Don’t present everything in your report!
- Key things to include
  - Objective
  - Preliminary result
  - Progress vs. schedule
- < 15 min presentation, 5 min for Q&A.
- Business casual attire
Modeling

- Linearization
- Comparison between experimental and (nonlinear) simulation results
State space approach to control design

State space model:

\[ \dot{x} = Ax + Bu + Gd \]

Consider the 1-axis model:

\[ \ddot{\theta} + a_1 \dot{\theta} + a_2 \text{sign} \dot{\theta} = a_3 u + a_4 \sin \theta. \]

Choose state variables as

\[ x_1 = \theta - \theta_{des}, \quad x_2 = \dot{\theta}. \]

Then

\[
\begin{bmatrix}
0 & 1 \\
-a_1 & -a_4 \cos \theta_{des}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
a_3
\end{bmatrix} u
+ \begin{bmatrix}
0 \\
a_4
\end{bmatrix} \sin \theta_{des}
- \begin{bmatrix}
0 \\
a_2
\end{bmatrix} \text{sign} \dot{\theta}.
\]
Full state feedback

Suppose \((x_1, x_2)\) are available (i.e., position and velocity), then a full state feedback control law is just the PD control:

\[
    u = -Fx = -F_1(\theta - \theta_{des}) - F_2\dot{\theta}.
\]

The closed loop system is

\[
    \dot{x} = (A - BF)x + Gd.
\]

The feedback gain \(F\) is chosen to place the closed loop poles (by using the MATLAB place or acker command). \((A, B)\) needs to be controllable.

Steady state error: \(x_{ss} = -(A - BF)^{-1}Gd\)
Removal of steady state error

To remove the steady state error, use $u$ to cancel the disturbance $d$, if possible.

If $d$ is unknown, or $Gd$ cannot be cancelled by $Bu$, apply integral control. Define the output that we’d like to remove the steady state error as

$$y = Cx = [1 \ 0]x.$$

Define a new state, $q$:

$$\dot{q} = y = Cx.$$

The state equation is then

$$\begin{bmatrix} \dot{x} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} G \\ 0 \end{bmatrix} d.$$
Integral feedback

The full state feedback control law is now just the PID control law:

\[ u = -F \begin{bmatrix} x \\ q \end{bmatrix} = -F_1(\theta - \theta_{des}) - F_2\dot{\theta} - F_3 \int (\theta - \theta_{des}). \]

The feedback gain now needs to be chosen to make the augmented closed loop system stable.

Only stability is assured, you still should check the loop shape versus disturbance rejection, trajectory tracking, and robustness (gain and phase margins).
What if full state is not available?

Given

$$\dot{x} = Ax + Bu + Gd, \ y = Cx.$$ 

We can estimate $x$ from $y$ by using a full state "observer" (Kalman filter is also of the same form). Construct a "replica" of the plant with a corrective feedback based on "innovation" (difference between measured output and predicted output):

$$\dot{x} = A\hat{x} + Bu + L(y - C\hat{x}).$$

Let $e = x - \hat{x}$ be the state estimation error, then

$$\dot{e} = (A - LC)e + Gd.$$ 

Now choose $L$ so that $A - LC$ is stable. But the eigenvalues of $A - LC$ is the same as $A^T - C^T L^T$. Therefore, use place or acker command to choose $L^T$ (based on $(A^T, C^T)$), and transpose the result.
Progress presentation tomorrow at 6pm in JEC 4304

team 5: 6:00, team 4: 6:20, team 2: 6:40,
team 1: 7:00, team 3: 7:20