Motion manifold is called an inherent motion. A motion along the manifold will always have joint velocities which
satisfy the condition 3.17. We obtain
\[ \dot{z} = \int (I(\theta) \theta) \theta \]
Although the geometric equations of a parallel manipulator are compl-
ected by the closed loop nature of the mechanism, the inverse problem
brarily formulates a Grüber's formula. For a parallel mechanism only
\[ f \sum_{j=1}^{N} (6-N_j)g_j = 0 \] (6.6)

the forward equations for a parallel manipulator are described by equa-
nor, the manipulator with its joints in the first chain (including the end-effector) and its end-effector location specified by each chain. Suppose we have a parallel manipulator in terms of the mechanism and location of the

parallel manipulators are also called closed-chain manipulators, since they contain

Figure 3.12: A parallel manipulator consisting of three serial chains con-

Figure 3.13: Self-motion manifold for a redundant planar manipulator.

The forward equation for a parallel mechanism

Figure 3.13: Self-motion manifold for a redundant planar manipulator.
describe the mechanism—the function of the links, shape of the chain, relative motion of the links, and so on. The choice of the kinematic parameters will depend on the specific problem and the type of mechanism desired. Many other problems exist, but for all problems through a set of points. Many other problems exist for all possible paths of motion on the mechanism. For example, one might wish to design a new type of mechanism for a given machine. Alternatively, one might wish to design a new functional relationship for a given mechanism.

To illustrate some of the concepts introduced above, we consider the following example:

**5.3 Four-bar linkage**

We study the Stewart platform in Section 5.4. The Stewart platform is a type of mechanism that can perform a wide variety of motions. We will consider the Stewart platform in this section, and we will see how the kinematics of the Stewart platform can be used to design mechanisms for other applications.

To describe the kinematics of the Stewart platform, we introduce the concept of a vector. A vector is a quantity that has both magnitude and direction. In this case, we will use vectors to represent the positions of the links in the chain. The position of each link is represented by a vector. The position of the first link is represented by vector \( \mathbf{r}_1 \), the position of the second link is represented by vector \( \mathbf{r}_2 \), and so on.

To describe the motion of the mechanism, we use the following equation:

\[
\mathbf{r}_A = \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4
\]

This equation has a form similar to the one we used earlier to describe the kinematics of a multi-linker. It is used to describe the position of the mechanism at any time. The right-hand side of the equation represents the sum of the vectors representing the positions of the individual links. The left-hand side represents the position of the mechanism as a whole. By taking the derivative of this equation with respect to time, we can determine the velocity of the mechanism. This is called the velocity equation:

\[
\mathbf{v}_A = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 + \mathbf{v}_4
\]

The velocity of the mechanism is the sum of the velocities of the individual links. By taking the derivative of this equation with respect to time, we can determine the acceleration of the mechanism. This is called the acceleration equation:

\[
\mathbf{a}_A = \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \mathbf{a}_4
\]

The acceleration of the mechanism is the sum of the accelerations of the individual links.

In Section 5.4, we will study the Stewart platform in more detail. We will see how the kinematics of the Stewart platform can be used to design mechanisms for other applications. We will also see how the kinematics of the Stewart platform can be used to design mechanisms for other applications.
The twist can be calculated using the formula for twists in the plane of a revolute joint.

The twist of an element in the plane of a revolute joint can be calculated using the formula:

\[ \theta = \frac{d}{r} \]

where \( \theta \) is the twist, \( d \) is the displacement, and \( r \) is the radius.

The twist of a revolute joint is given by the formula:

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where \( \theta \) is the twist, \( d \) is the displacement, and \( r \) is the radius.
In order to write the structure equation for a Stewart platform, we
complete the Sylvestrian (or linear) equations for a Stewart platform have been solved in
the form of (12) in Section 2.1.2. It is only recently
described by Stewart in the 1959 paper [19]. Although the concept of a Stewart platform is
an interesting and desirable concept, the motion involved is quite complex. The
motion can be divided into two independent motions: the linear and rotational
motion.

Stewart platforms are commonly used in many different applications, such as
robotic manipulators, flight simulators, and other systems where precise control is
required.

Another common example of a Stewart platform is the Stewart platform used in
robotic arms.

5.4 Stewart Platform

and hence uncertainty configurations usually occur in practice.

![Stewart Platform Diagram](image_url)

Figure 3.2: A Stewart platform with a Puma robot attached. (Photo)

The configuration shown in Figure 3.2 is known as an uncertainty
configuration in the Stewart platform. In this case, it is actually
possible for the mechanism to move in uncertainty in two independent
configurations. An example of a Stewart platform is shown in Figure 3.2. Note that this
configuration is not shown in the actual system because it is not
practical. However, it is shown in Figure 3.2 to illustrate the
concepts of Stewart platforms.

The right-hand side of equation (3.17) corresponds to the mass-
gravity. This means that if the passive joints are collinear, then the
right-hand side of equation (3.17) is nonlinear.

The right-hand side of equation (3.17) is nonlinear.

![Uncertainty Configuration](image_url)

(a) Uncertainty configuration

(b) Stewart configuration

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gravity. This means that if the passive joints are collinear, then the
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The right-hand side of equation (3.17) corresponds to the mass-
gravity. This means that if the passive joints are collinear, then the
right-hand side of equation (3.17) is nonlinear.
The problem of inverse kinematics for a Stewart platform is a very different problem due to the increase in the number of degrees of freedom in the platform. The Stewart platform is a very flexible structure that can be used in many different applications. The model of the Stewart platform is given by the equation:

\[
\begin{bmatrix}
\mathbf{e}_1 \\
\mathbf{e}_2 \\
\mathbf{e}_3
\end{bmatrix} = \mathbf{T}(\theta)
\]

where \( \mathbf{T}(\theta) \) is the transform matrix of the Stewart platform.

The Jacobian matrix \( \mathbf{J} \) is given by:

\[
\mathbf{J} = \begin{bmatrix}
\frac{\partial \mathbf{e}_1}{\partial \mathbf{q}} & \frac{\partial \mathbf{e}_2}{\partial \mathbf{q}} & \frac{\partial \mathbf{e}_3}{\partial \mathbf{q}}
\end{bmatrix}
\]

where \( \mathbf{q} \) is the vector of joint angles.

The equation of the constraint is:

\[
\mathbf{J} \mathbf{q} = \mathbf{0}
\]

The motion is constrained by a special joint form.

The special solution is:

\[
\mathbf{q} = \mathbf{K} \mathbf{e}
\]

where \( \mathbf{K} \) is the null space of \( \mathbf{J} \).

More useful frameworks are present as:

\[
\mathbf{A} \mathbf{q} = \mathbf{b}
\]

where \( \mathbf{A} \) is a free parameter (the vector). To cast this equation into a form where \( \mathbf{b} \) is a free parameter, the vector and \( \mathbf{b} \) is the location of the center of the platform.
\[
\begin{bmatrix}
\alpha_a \\
0 \\
0 \\
1_a \\
0
\end{bmatrix} \begin{bmatrix}
0 & 0 & 0 \\
0 & \alpha_a & 0 \\
\alpha_a & 0 & 1_a \\
1 & 0 & 0
\end{bmatrix} = \phi
\]

In this configuration, it is not possible for the mechanism to rotate around the intersection point. If we write down the wrenches relative to a coordinate frame centered at the intersection point, it is clear that we cannot generate a force exerted by the actuator. It is clear that we cannot generate a torque around the intersection point since we're in the prismatic axis and it is the force where it is the direction of the wrench diagram.
be generalized by the actual joint's drops rank.

where the dependence on the set of end-effector forces which can satisfy the condition of the space of admissible forces drops rank. Other

when Θ = Θ \( \Theta^{(i)} \) where

\[
\Theta^{(i)} \ni x^2 \nabla f = \Theta \nabla f = I \nabla f = I v_f^\perp
\]

of the structure equation has the form

where if is twist for the end-effector chain. The Jacobian

\[
(0)^{n\theta_2} \omega_{\theta_2} \omega_{\theta_3} \ldots \omega_{\theta_{n\theta}} = (0)^{n\theta_2} \omega_{\theta_2} \omega_{\theta_3} \ldots \omega_{\theta_{n\theta}} = I v_f^\perp
\]

satisfies the structure equation.

7. A parallel manipulator has multiple kinematic chains connecting the

\[
\theta(0) = 0 = \theta(\theta) \nabla f
\]

in motion above the self-motion manifold and satisfy

whose configuration of the end-effector. External motions correspond

describe the set of joint values which can be used to describe a ds-6

self-motion manifold minimally required degrees of freedom. The self-motion manifold

6. A manipulator is kinematically redundant if it has more than the

to singularity.

The manipularity of a robot provides a measure of the nearness

of one point to another

intersection exists

Subproblem 3: Intersection exists

\[
g = ||d_\theta^\perp - b||
\]

Subproblem 2: More about two

neighbor another

\[
b = d_\theta^\perp
\]

Subproblem 1: More about one point onto

inverse kinematics by making use of the following subproblems:

2. The (complete) workspace of a manipulator is the set of end-effector

where \( \Theta = \Theta^{(i)} \) and the joint values \( \Theta \) to the end-effector

applicable set of subproblems.

To find a complete solution, we apply the manipulator kinematics.

1. The forward kinematics of a manipulator is described by a mapping

The following are the key concepts covered in this chapter:
Exercises

8

Figure 3.23: Some simple three degree of freedom manipulators.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

(m)

(n)

(o)

(p)

(q)

(r)

(s)

(t)

(u)

(v)

(w)

(x)

(y)

(z)

1. Find the forward kinematics map.
2. For each of the manipulators shown schematically in Figure 3.24:
   (a) Derive the spatial and body Jacobians.
3. Solve the Inverse Kinematics problem using the Paden-Kahan
   subproblems.
4. Find the forward kinematics map.
5. Find the forward kinematics map.

1. Draw the twist axes for the manipulators shown in Chapter 1.

Bibliography

There is a vast literature on robot kinematics, including a number of text-

explosions on kinematics and design of mechanisms.
9. Singular values of a matrix

8. Show that the spatial velocity of a manipulator does not depend on the location of the tool frame (as long as it moves with the base).

7. Suppose the 5: Translation to a given distance

6. Problem 4: Rotation about two non-intersecting axes

5. Problem 2: Rotation about two non-intersecting axes

4. Example 5: The number of inverse kinematics solutions in different configurations

3. Describe the reachable and dextrous workspaces and calculate the joint limits.

2. Give a geometric description of the singular configurations.

1. Solve the special and body Jacobians.

Suppose the inverse kinematics problem is the Paden-Kahan
Syntactic parallel joints are represented by rectangular boxes.

Figure 3.14: Sample manipulators. Revolute joints are represented by disks.

Figure 3.15: Rhino robot

Figure 3.2: Subtract manipulator

Figure 3.3: Add manipulator

Figure 3.4: Sample manipulators. Revolute joints are represented by circles.

Figure 3.5: Rhino robot

Figure 3.6: Subtract manipulator

Figure 3.7: Add manipulator

Figure 3.8: Sample manipulators. Revolute joints are represented by rectangles.

Figure 3.9: Rhino robot

Figure 3.10: Subtract manipulator

Figure 3.11: Add manipulator

Figure 3.12: Sample manipulators. Revolute joints are represented by squares.

Figure 3.13: Rhino robot

Figure 3.14: Subtract manipulator

Figure 3.15: Add manipulator

Figure 3.16: Sample manipulators. Revolute joints are represented by triangles.

Figure 3.17: Rhino robot

Figure 3.18: Subtract manipulator

Figure 3.19: Add manipulator

Figure 3.20: Sample manipulators. Revolute joints are represented by diamonds.

Figure 3.21: Rhino robot

Figure 3.22: Subtract manipulator

Figure 3.23: Add manipulator

Figure 3.24: Sample manipulators. Revolute joints are represented by pentagons.

Figure 3.25: Rhino robot

Figure 3.26: Subtract manipulator

Figure 3.27: Add manipulator

Figure 3.28: Sample manipulators. Revolute joints are represented by hexagons.
such that
are said to be coplanar if there exists a plane with unit normal \( n = (a, b, c) \). For a regular surface, the Gauss map is defined as

\[ \text{Gauss map} = \frac{\partial \mathbf{r}}{\partial s} \times \frac{\partial \mathbf{r}}{\partial t} \]

12. \text{Homogeneous singularities:} Your coplanar regular points

(c) \( \text{XZ Plane only:} \ X \neq 0 \)
(b) \( \text{YZ Plane only:} \ Y \neq 0 \)
(a) \( \text{XY Plane only:} \ Y \neq 0 \)

13. \text{Stability for the following class of singular points:}

(c) \text{Diagonal with isotropic points are useful for those which involve}

(a) \text{Close to a vertex.}
(b) \text{Close to a loop line in the plane.}

14. \text{Condition number of the Jacobian is 1.}

15. \text{Isotropic points:}

- \( \sum \frac{\partial^2}{\partial s^2} \), \( \sum \frac{\partial^2}{\partial t^2} \), \( \sum \frac{\partial^2}{\partial s \partial t} \)
- \( \frac{\partial }{\partial s} \), \( \frac{\partial }{\partial t} \)
- \( \text{characterized in terms of the singular values of } \theta \)

\[ \left\{ \theta \in (0, \pi) \mid R \text{ is max} = (\theta)^n \right\} \]

Define a manipulability measure on \( \theta \)

\[ \theta \geq \left( \frac{\partial \theta}{\partial \theta} \right) + \sum \epsilon \left( \frac{\partial \theta}{\partial \theta} \right) : R = \theta^n \]

16. \text{Let } \{ \theta \} \Rightarrow \text{be the Jacobian of a manipulator (a)}

\[ \theta > \left( \frac{\partial \theta}{\partial \theta} \right) + \sum \epsilon \left( \frac{\partial \theta}{\partial \theta} \right) : R = \theta^n \]

\[ \text{The task requirement is taken into account.} \]

\[ \text{With the principal axes of length } (a, b, c) \text{ oriented as}\]

\[ \text{Assume that a task is modeled by an elliptical in the task space of } \theta = d \text{.} \]

17. \text{Let } \{ \theta \} \Rightarrow \text{be the Jacobian of a manipulator (b)}

\[ \text{Where } A, \text{ and } I \text{ are partitioned as}\]

\[ \text{So that the columns of } A, \text{ and } I \text{ are partitioned as}\]

\[ \text{Where } A, \text{ and } I \text{ are partitioned as}\]
Consider the slider-crank mechanism shown below:

where the components of the Jacobian:

19. Show that if a manipulator is at a singular configuration, then there

18. In general, the manipulator Jacobian depends on the choice of base and/or

17. Kinematic singularity: Prismatic joint perpendicular to two parallel

16. Kinematic singularity: Six revolute joints intersecting along a line

15. Show that when the six revolute joint axes are coplanar, a six-degree-of-freedom

14. prismatic joint perpendicular to two parallel joints.


12. In general, the manipulator Jacobian depends on the choice of base.

11. Kinematic singularity: prismatic joint perpendicular to two parallel joints.

10. Show that if a manipulator is at a singular configuration, then there

9. Show that if a manipulator is at a singular configuration, then there

8. Show that if a manipulator is at a singular configuration, then there

7. Show that if a manipulator is at a singular configuration, then there

6. Show that if a manipulator is at a singular configuration, then there

5. Show that if a manipulator is at a singular configuration, then there

4. Show that if a manipulator is at a singular configuration, then there

3. Show that if a manipulator is at a singular configuration, then there

2. Show that if a manipulator is at a singular configuration, then there

1. Show that if a manipulator is at a singular configuration, then there

Conclusions: Under what conditions (a) is the active set of terms used?

Active set:

Find the singular configuration of the mechanism.

Cylindrical coordinates are used in the active set.

Calculate the structure equations for the mechanism.

Calculate the number of degrees of freedom of the mechanism.

Example of a manipulator exhibiting such a singularity.
Chapter 4
Robot Dynamics and Control