Kinematic calibration of the parallel Delta robot
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SUMMARY
This article deals with the kinematic calibration of the Delta robot. Two different calibration models are introduced: The first one takes into account deviations of all mechanical parts except the spherical joints, which are assumed to be perfect (“model 54”), the second model considers only deviations which affect the position of the end-effector, but not its orientation, assuming that the “spatial parallelogram” remains perfect (“model 24”). A measurement set-up is presented which allows to determine the end-effector’s position and orientation with respect to the base. The measurement points are later used to identify the parameters of the two calibration model resulting in an accuracy improvement of a factor of 12.3 for the position and a factor of 3.7 for the prediction of the orientation.

KEYWORDS: Delta robot; Kinematic calibration; Two calibration models; Implicit calibration.

1. INTRODUCTION
Parallel mechanisms are generally regarded as being highly accurate due to the non-cumulative joint errors. Programming of high precision assembly tasks by the traditional “teach-in” method becomes very expensive. Hence, off-line programming is needed which claims a robot with a small static pose error, or in other words, with a high accuracy. The accuracy of parallel robot can be improved by an appropriated calibration technique, which is the subject of this paper.

The major part of articles addressing calibration methods for parallel robots are based on the Stewart\(^2\) Platform which is a fully parallel non-redundant manipulator with six degrees of freedom (DOF). Many authors\(^3-6\) used a calibration model assuming universal (U-) and spherical (S-)joints to be perfect as well as prismatic (P-)actuators to be perfectly assembled, which leads to a model with 42 kinematic parameters (“model 42”). Not assuming U, S and P-joints to be perfect would lead to 138 kinematic parameters.

However, some of the problems encountered in improvement of accuracy of parallel robots cannot be demonstrated with the Stewart Platform. That is:

a) Model 42 can be established without the risk of introducing mathematical singularities in the parameterization. The 36 parameters used to describe the Cartesian coordinates of the attachment points of the U- and S-joints are free of singularities. The remaining 6 parameters, the transducers’ off-set, describing a distance are free of singularities, too. The modeling process is simple due to the lack of rotative (R-)joints, the joint axis of which has to be modeled, and due to the very simple topology of the kinematic chains (“legs”) which link the end-effector to the base.

b) Some errors in the pose of the end-effector which occur for manipulators with less than 6 DOF cannot be influenced by its actuators. These non-influenceable errors are imposed by the mechanical structure of the robot and cannot be corrected by a calibration procedure. Taking for instance a SCARA robot with its 4 DOF, it is obvious that the remaining, non-influenceable 2 DOF correspond to the perpendicularly of the end-effector with respect to its base.

A more sophisticated test vehicle for the calibration of parallel robots may be the Delta robot\(^7\) with its 3 translational DOF and rotative actuators (Figure 1). Its kinematic chains as well are more complex than the “legs” of a Stewart Platform, since they are composed of an “arm” which branches into two parallel “forearms”. Furthermore, calibration is of particular interest since this robot is marketed by Demaurex, Robotique & Microtechnique S.A.

The Delta robot maintains passively its end-effector’s orientation with respect to the base. The concept was termed “spatial parallelogram”. For calibration this particularity of the Delta robot introduces kinematic parameters which are nearly unobservable. If for a calibration model the spatial parallelogram is assumed to be perfect, the end-effector size as well as the distance between the two parallel forearms become totally unobservable and therefore unidentifiable. However, small mechanical deviations will disturb the spatial parallelogram and cause small changes of the orientation. The observability of the kinematic parameters mentioned above (end-effector size and distance between the forearms) will still be very small compared to other parameters such as the length of the arms for instance.

The aim of this paper is to present an experimentally verified calibration of the Delta robot using the methods developed in Vischer.\(^8\)

According to Mooring\(^9\) a calibration process consists of four different steps: modeling, measurement, identification and implementation. This paper is structured accordingly. Existing work on the calibration of the Delta robot is first reviewed.
2. SURVEY OF LITERATURE

The first approach to calibrate a Delta robot was made by Zobel. Based on the assumption that the end-effector remains perfectly parallel to the base, he proposed a calibration model containing 18 parameters. To identify these parameters a premeasured fixture (precision plate) with six touch points for full position measurement was used. The fixture could be placed in three different positions on the base plate of the robot. In a first experiment 3 parameters were identified using 3 error equations and it was shown that an error of 10 millimeters could easily be identified. For verification the 3 parameters have also been directly measured. However, the introduced test-error of 10 millimeters is large (5%) compared to the characteristic length of the robot (length of the arm: 205 mm). Standard manufacturing tolerances of such a piece are in the range of 0.5–0.01 millimeters (0.25–0.005%).

Mauroine proposed a recalibration procedure for a Delta robot based on a displacement measurement of a single Laser sensor (triangulation). He stated that a calibrated robot moved to a new work place has to be recalibrated with respect to its environment and that the offsets of the joint transducers must be reidentified. To identify 9 parameters he proposed a two-step method. In a first step a plane is precisely located parallel to the base plate and a first set of 6 parameters is identified with this set-up. In a second step small cylinders are arranged in a circle on this plane and the remaining three parameters identified. Simulations were performed with 200 to 600 micrometers of measurement noise in order to show the robustness of the proposed method. For the experimental part of his work the orientation of the plane as well as the location of the cylinders were identified in a previous step. Different sets of parameters were identified depending on the initial values for the iterative non-linear least square algorithm (problem of multiple minima).

Lintott simulated the calibration procedure of a Delta robot by investigating in a first step a Stewart Platform. He stated that the lower part (sub-structure) of the Delta robot (Figure 1) represents a general Stewart Platform when subjected to mechanical errors. He investigated the optimal choice of measurement points and observed that they migrate towards the edges of the workspace (inverse singularities) and towards the singular configurations within the workspace (direct singularities). However, measurement points located within singularities will cause problems during the identification phase. He adapted the method developed for the calibration of serial robots which requires solving of the direct problem (forward calibration, Figure 9) of the calibration model during identification. This is very time-consuming. Based on simulated noisy measurement data and using the Levenberg-Marquardt algorithm for non-linear least-squares estimation, the Euclidean norm of the error vector in the position could be improved by a factor of 50 (4.33 mm $\rightarrow$ 0.086 mm). For the orientation he reached an improvement factor of 417 (3.1 degrees $\rightarrow$ 27 arcseconds). Such high improvement factors are difficult to reach in an experimental calibration as opposed to a simulated one. As a rule of thumb the accuracy of the measurement unit must be a magnitude higher than the level which should be gained by calibration. Using current technology it is difficult and expensive to provide a 3D-orientation measurement unit with an accuracy of 2.7 arcseconds.

3. MODELING

A “good” calibration model must fulfill three criteria: completeness, equivalence, and proportionality.
- A complete model contains a sufficient number of parameters to describe the mechanical structure of a robot without being redundant. According to Vischer this number of independent parameters (C) can be calculated for a multi-loop parallel robot as follows:

\[ C = 3R + P + SS + E + 6L + 6(F - 1) \]  

In addition to open-loop structures a multi-loop mechanism with L loops may contain unsensed spherical joints (S) as well as revolute (R) and prismatic joints (P), which can be either sensed or unsensed. SS counts the number of pairs of S-joints. The number of measurement transducers is counted by E whereas F is the number of arbitrarily located frames. F typically equals two for an arbitrarily located base frame {B} and moving frame {P}.
- A parameterization is proportional if small changes in the geometry of the robot are reflected by small changes in the parameters. Thus, proportionality addresses the problem of mathematical singularities, which can be introduced if not choosing carefully the parameterization. The classical example is the DH-(Denavit & Hartenberg) parameterization, which fails to be proportional for nearly parallel joint axes. According to Hayati problems of unproportionality can be avoided by taking for nearly parallel joint axes the H-(Hayati) parameterization and for nearly perpendicular axes the DH-parameterization.
- Two calibration models are equivalent if they are complete and proportional.

A calibration model for the Delta robot considering all possible geometrical deviations would have 138 parameters. This can be verified by equation (1), where an
S-pair is modeled as a 5R-joint-link train. \((R = 33, E = 3, L = 5, F = 2)\). Based on a simulation of a Stewart Platform, Wang\(^8\) concluded that errors in passive multi-DOF joints are negligible compared to other manufacturing errors. Furthermore, special design efforts were made in Vischer\(^6\) to create S-joints which are as perfect as possible. If the S-joints are modeled as perfectly, the number of parameters drops to 54 \((R = 3, SS = 6, E = 3, L = 5, F = 2)\). This calibration model will be referred to as “model 54”. Assuming further that the “spatial parallelogram” remains perfect, model 54 can be reduced to a model containing 24 parameters, which is termed “model 24”. In order to achieve proportionality special care has to be taken about three nearly parallel lines, which are the motor axis, the connecting line of the proximal and of the distal S-joints (Figure 2).

3.1. Parameterization

For better understanding of the parameterization the Delta mechanism is first shown without geometric deviations of its mechanical parts (Figure 2, nominal robot) whereas in Figure 3 deviations are introduced. The upside-down representation corresponds to the measurement set-up shown in Figure 6.

For parameterization the end-effector is considered to be fixed to the base-plate (Figure 3). This allows to parameterize model 24 and model 54 using 24 identical parameters.

According to Figure 2 and 3 the following points, lines and frames are defined:

\(B_{i,1–2}\): Center points of the S-joints attached to the end-effector→distal S-joints

\(C_{i,1–2}\): Center points of the S-joints attached to the arms→proximal S-joints

\(\ell_i, \ell_i':\) Connecting straight section from \(B_{i,1}\) to \(B_{i,2}\) and \(C_{i,1}\) to \(C_{i,2}\), respectively

\(B_i, C_i:\) Mid-point of the section \(\ell_i\) and \(\ell_i'\), respectively

\(O_i:\) Projection point of \(C_i\) on the motor axis

\(O_{i,1–2}:\) Points on the motor axis located at a distance of \(\pm \ell_i/2\) of \(O_i\)

\{B\}: The base frame \{B\} is arbitrarily fixed to the base

\{P\}: The moving frame \{P\} is arbitrarily fixed to the end-effector

\{0\}: The z-axis of the distal S-joint frame \{0\} is parallel to \(\ell_p\)

\{1\}: The z-axis of the motor frame \{1\} is parallel to the motor axis

\{2\}: The twisted motor frame \{2\} is the frame \{1\} twisted by the motor angle

\(6\) world coordinates

\(\mathbf{n} \mathbf{P} = [x, y, z]^T\)

\(\mathbf{n} \mathbf{R} = \text{Rot}(z, \gamma) \cdot \text{Rot}(y, \beta) \cdot \text{Rot}(x, \alpha)\)

\(3\) joint coordinates \(i = 1 \cdots 3\)

\(\mathbf{q} = \text{Rot}(z, \theta_i)\)

\(54\) kinematic parameters \(i = 1 \cdots 3\)

\(\mathbf{n} \mathbf{T}_i = \mathbf{n} \mathbf{T}_i = \text{Rot}(z, \theta_i) \cdot \text{Rot}(x, \alpha_i)\)

\(H\)-parameters for nearly parallel axes:

\(\mathbf{h} \mathbf{T}_i = \text{Rot}(x, \Delta \alpha_i) \cdot \text{Rot}(y, \Delta \beta_i)\)

Fig. 2. Delta mechanism without deviations.
Vector from \( B_i \) of the attached end-effector to the point \( O_i \) on the motor axis:
\[
\begin{align*}
{\mathbf{d}}_i &= (D_{x_i}, D_{y_i}, D_{z_i})^T \\
\end{align*}
\]
Vector \( O_i \) to the \( C_i \) including the encoder offset \( \theta O_i \) and the arm length \( L a_i \):
\[
\begin{align*}
{\mathbf{L}a}_i &= (L a_{x_i}, L a_{y_i}, 0)^T \\
\end{align*}
\]
Vector from the origin of \( \{P\} \)-frame to \( B_i \):
\[
\begin{align*}
{\mathbf{b}}_i &= (b_{x_i}, b_{y_i}, b_{z_i})^T \\
\end{align*}
\]
Vector whose \( z \)-component is half as long as section \( \mathcal{L}_d \):
\[
\begin{align*}
{\mathbf{d}}_i &= (0, 0, d_{z_i})^T \\
\end{align*}
\]
Error vector: Difference between vector \( \mathbf{OC}_{i,1} (= C_{i,1} - O_i) \) and vector \( \mathbf{OC}_i (= C_i - O_i) \):
\[
\begin{align*}
{\Delta \mathbf{C}}_i &= (\Delta C_{x_i}, \Delta C_{y_i}, \Delta C_{z_i})^T \\
\end{align*}
\]
Average length of the forearms:
\[
L a_i \\
\]
Half of the difference of the forearms' lengths:
\[
\Delta L a_i \\
\]
These are the scalars, vectors and rotation matrices, which together parametrize the Delta robot completely.

3.2. Model 54
As model 42 of the Stewart Platform, model 54 must also satisfy six closure equations since the upper part of a Delta robot (the 6 forearms and the end-effector) is an immobile Stewart Platform (Figure 2). These six closure equations will be coupled in pairs in the three joint coordinates (motor angles). Such a pair of closure equations represents one of the three main joint-link trains (Figure 3). Since model 54 will later on be reduced to model 24, it is more convenient to describe one main joint-link train by the sum \( G1 \) and the difference \( G2 \) of these two closure equations. For simplicity the leading sub- and superscripts are dropped yields model 54:

\[
\begin{align*}
G1: & \quad \mathbf{C}B_i^T \cdot \mathbf{C}B_j + \Delta \mathbf{d}_i^T \cdot \Delta \mathbf{d}_i = L a_i^2 + \Delta L a_i^2 \\
G2: & \quad \mathbf{C}B_i^T \cdot \Delta \mathbf{d}_i = L a_i \cdot \Delta L a_i \\
\end{align*}
\]

\[
\begin{align*}
\mathbf{C}B_i = & \quad \mathbf{P} + \mathbf{R} \cdot \mathbf{T}_i \cdot \mathbf{b}_i - \mathbf{T}_i \cdot (\mathbf{b}_i + \mathbf{D}_i + \Delta \mathbf{T}_i \cdot \mathbf{Q}_i \cdot \mathbf{L}a_i) \\
\Delta \mathbf{d}_i = & \quad \mathbf{R} \cdot \mathbf{T}_i \cdot \mathbf{d}_i - \mathbf{T}_i \cdot \Delta \mathbf{T}_i \cdot (\mathbf{d}_i + \mathbf{Q}_i \cdot \Delta \mathbf{C}_i) \\
\end{align*}
\]

\[
i = 1 \cdots 3 \\
\]

3.3. Model 24
To establish model 24, model 54 is simplified by assuming that the end-effector remains perfectly parallel to the base frame. In other words: The spatial parallelogram is modeled as being perfect:

\[
\mathbf{R} = \mathbf{I} \quad \text{where} \quad \mathbf{I} \text{ is the } 3 \times 3 \text{ identity matrix} \\
\]

The number of world coordinates (equation (2)) is reduced from 6 to the 3 Cartesian coordinates describing the origin of the \( \{P\} \)-frame. This simplification is only valid if the three lines given by the axis of the motor, the connecting section of the proximal S-joints and the connecting section of the distal S-joints remain perfectly parallel to each other. In order to reach this 18 parameters are fixed on their nominal values.

\[
P1: \Delta \mathbf{T}_i = \mathbf{I}, \quad \Delta \mathbf{C}_i = \mathbf{0}, \quad \Delta L a_i = 0 \quad i = 1 \cdots 3 \\
\]
Substituting equation (6) and equation (7) into equation (5) shows that the $\mathbf{b}_i$, as well as the $\mathbf{d}_i$, Vector containing altogether another 12 parameters vanish.

$$ P2: \mathbf{b}_i, \mathbf{d}_i \rightarrow \text{vanish} \quad i = 1 \cdots 3$$

Geometrically, this corresponds to the reduction of the end-effector to a single point and the degeneration of the $R(2S/2S)$ joint-link train to a $R2S$ chain as shown in Figure 4.

A further consequence of equation (8) is the degeneration of the second set of equations $G2$ given in equation (5) to identity ($0 = 0$) whereas the first set $G1$ leads to model 24:

$$ \mathbf{C}^2_{\mathbf{B}} \cdot \mathbf{C}_i = \mathbf{L} \mathbf{b}_i^2$$

with

$$ \mathbf{C}_i = \mathbf{P} - \mathbf{T}_i \cdot (\mathbf{D}_i + \mathbf{Q}_I \cdot \mathbf{L}_a)$$

This model can be applied to the Delta robot assuming that its end-effector always stays perfectly parallel to the base, which corresponds to assumption for the nominal model of Clavel. It can therefore be said that model 24 is an extended nominal model.

3.4. Conclusion for model 24 and 54

Reducing model 54 to model 24 leads to the following characteristic properties of model 54:

a) The first set of closure equations $G1$ contains model 24, whereas the second set $G2$ will degenerate to identity (equation (5)).

b) Only 30 of the 54 parameters have an influence on the orientation of the end-effector. These 30 parameters could further be split into two subsets:

c) The generating set $P1$ contains 18 parameters of magnitude $\Delta$ reflecting small errors in the joint-link train (equation (7))

d) The amplifying set $P2$ contains 12 parameters of magnitude 1 describing the dimensions of the end-effector and the distance between the forearms (equation (8)).

These two sets are related in an interesting way: If $P1$ is considered to be zero, $P2$ has no influence on the pose of the end-effector. The set $P2$ cannot generate pose errors by itself, but if $P1$ is not zero, it will amplify them. In Figure 5 this is represented symbolically by a triangle.

It can further be concluded:

- Variation of $P1$ affects the orientation much more than variation of $P2$. Thus, $P1$ is much more sensitive to orientation errors than $P2$. To build Delta robots with smallest possible orientation errors efforts have thus to be concentrated on the 18 parameters of set $P1$.

- A parameter set which causes small end-effector errors over the whole workspace is nearly unobservable in the identification phase. For identification the parameters of set $P2$ will be fixed to their nominal values and only the remaining 42 parameters will be identified.

- The bigger $P2$ becomes, the smaller is the orientation error. This is useful for the choice of the nominal parameters for an application of the Delta robot.

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Fig. 4. Geometric interpretation of model 24 as a spatial 3[R2S] structure.
aiming at being very precise. In this case $P2$ should be chosen as large as possible with respect to the remaining parameters. A larger distance between the forearms for instance will decrease the resulting orientation error of the end-effector.

4. MEASUREMENT

The principle goal of the measurement step is to gain some redundant information about the robot to be calibrated. Measurement set-ups can be grouped into classes, depending on whether external measurement devices are used or not:

Set-ups without external measurement devices are very dependent on the robot’s topology. They provide only partial information on the end-effector’s pose (e.g. when sliding on a plane). For parallel robots this is fatal since the direct problem of the calibration model has to be solved (in order to substitute $x, y, z$ in the equation of the plane). This results not only in a cumbersome mathematical treatment, but also the problem of multiple minima arises.

Set-ups with external measurement devices are more expensive since external sensors are added, but they offer the advantage that the end-effector’s full pose (position and orientation) can be measured. By substituting these measurement points into the 3 pairs of closure equations of a main joint-link train, the pairs become decoupled and there is no need to solve the direct nor the inverse problem.

Figure 6 shows our full pose measurement set-up for the Delta robot. Its base is rigidly attached to a 3D measuring machine (TESA Validator 10) whereas the end-effector is fixed to the $z$-axis of the measuring machine by means of a spherical joint (Figure 7). The accuracy of the measuring machine is ±10 micrometers and the measurement volume $300 \times 300 \times 120$ millimeters.

The end-effector is able to twist about the spherical joint when the orientation changes with respect to the base. Three linear digital probes (TESA-GT22C) orthogonally arranged to each other are used to measure the orientation with an accuracy of ±15 arcseconds (within 1.9 degrees). The measurement volume is ±4.7 degrees for each of the three angles.

The joint angles are measured by high resolution Laser encoders (CANON M1) with an accuracy of 25 arcseconds (accumulated error per revolution).

With the full-pose measurement set-up shown in Figure 6 a set of 74 measurement points was acquired, which are about uniformly distributed within the workspace. This set is used in the next section for identification of the kinematic parameters.

5. IDENTIFICATION

Identification is the central step of calibration. The parameters of the calibration model are determined to match the measurement data most closely.

Based on simulations of the calibration of a single-loop structure, implicit calibration (Figure 8) was proposed as the standard calibration method for parallel mechanisms.
5.1. Implicit calibration
Implicit calibration of parallel robots starts from the closure equations. These equations are generally coupled in the world coordinates and decoupled in the joint coordinates and the kinematic parameters. (Only valid for not too complicated models such as model 24 and model 42. In contrast, model 54 is also pairwise coupled in the joint coordinates as well as the kinematic parameters.) Measuring all joint coordinates and especially all world coordinates the identification problem becomes decoupled for each chain. Taking for instance model 24 (equation (9)) the residual \( r_j \) of one of the three main chains can be written as:

\[
\begin{align*}
  r_j &= CB^T \cdot CB - Lb^2 \\
  &\text{with } j = 1 \cdots N
\end{align*}
\]

Equation (10) together with equation (11) yields a non-linear least-squares estimation problem since the kinematic parameters are contained in the model (equation (10)) in a non-linear way. Furthermore, the tolerances allocated to the mechanical parts are ignored, which allows to treat the estimation problem as unconstrained. It can be solved with the Levenberg-Marquardt (LM-) algorithm\(^1\) which is implemented in the "optimization toolbox" of MatLab\(^3\). The LM-algorithm is a mixture between the Gauss–Newton and the steepest descent algorithm aiming at keeping the quadratic convergence rate of the Gauss–Newton algorithm by avoiding the problem of rank deficiency of the identification Jacobian.

The nominal parameters of the Delta robot shown in Figure 6 are listed in Table I. They are used as initial values for the iterative LM-algorithm.

- **Identification of model 24**
For the identification of the 8 parameters \( n = 8 \) of one of the three main chains (equation (10)) of model 24 (equation (9)) only the position of the end-effector and joint angle are required from the 74 measurement \( (N = 74) \) points collected. The \( 74 \times 8 \) identification Jacobian needed for the LM-algorithm can be analytically differentiated. The eight partial derivatives consist of only 61 different factors.

With \( 3 \times 9 \) iterations the LM-algorithm has identified the following parameter set (Table II) in \( 3 \times 29 \) seconds (Pentium processor running at 90 MHz) using the nominal parameters given in Table I as initial guess. In contrast to in here discussed implicit calibration (Figure 8), forward calibration (Figure 9) with a numerically
Table I. Nominal parameters of the Delta robot

<table>
<thead>
<tr>
<th>main chain</th>
<th>$D_x$</th>
<th>$D_y$</th>
<th>$D_z$</th>
<th>$\vartheta$</th>
<th>$\alpha$</th>
<th>$La_x$</th>
<th>$La_y$</th>
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<tr>
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<td>119.963</td>
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<td>240</td>
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<td>76</td>
<td>-16.5</td>
<td>0</td>
<td>0</td>
<td>240</td>
<td>119.963</td>
<td>-3</td>
<td>240</td>
</tr>
</tbody>
</table>

additional 30 parameters for model 54

<table>
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<tr>
<th>main chain</th>
<th>$\Delta \alpha$</th>
<th>$\Delta \beta$</th>
<th>$\Delta C_x$</th>
<th>$\Delta C_y$</th>
<th>$\Delta C_z$</th>
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<td>20</td>
<td>24</td>
<td>0</td>
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Fig. 10. Position error of the Delta's end-effector before (dashed line) and after (solid line) calibration of model 24 (equation (9)).

Yields to a quite reliable calibration. This rule of thumb of taking twice as much measurement points as parameters of the model (one main chain has 8 kinematic parameters) is also supported by the work of Zhuang.3

- Identification of model 54

As shown in the modeling section, 12 parameters (set P2, equation (8)) of model 54 will be set to their nominal values since they are much less likely to cause pose errors than the remaining 42 parameters. The entire measured pose of the 74 collected measurement points is needed, including the measured deviations of the parallelism between the end-effector and the base.

Implicit calibration was performed by splitting the entire problem into three subproblems. Thus, each main joint-link train forming a double-loop structure together with the measurement device was identified separately. In $3 \times 13$ iterations using $3 \times 740$ seconds of calculation time, the LM-algorithm has identified the parameter set shown in Table IV using the nominal parameter set (Table I) as an initial guess.

The improvement of the end-effector’s position is comparable to the result of model 24 (Figure 10). Furthermore, the calibrated model 54 allows a better prediction of the end-effector’s orientation. In Figure 12 the norm of the error vector of the orientation is shown. It corresponds to the difference between the measured values and the values calculated based on model 54.

Table II. Identified parameters of model 24 (equation (9))

<table>
<thead>
<tr>
<th>main chain</th>
<th>$D_x$</th>
<th>$D_y$</th>
<th>$D_z$</th>
<th>$\vartheta$</th>
<th>$\alpha$</th>
<th>$La_x$</th>
<th>$La_y$</th>
<th>$Lb$</th>
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<tr>
<td>3</td>
<td>76.023</td>
<td>-16.624</td>
<td>0.062</td>
<td>239.998</td>
<td>90.010</td>
<td>119.990</td>
<td>-2.672</td>
<td>239.949</td>
</tr>
</tbody>
</table>
The standard deviation doesn’t decrease a lot since it is very difficult to measure the orientation of a rigid body in space with sufficient accuracy.

However, the mean values have improved by a factor of 12 in the position and 3.7 in the orientation, which proofs that implicit calibration works well for parallel robots.

5.2. Semiparametric calibration

By expanding the closure equations and replacing each coefficient in front of a combination of joint and/or world coordinates by a linear independent factor, the calibration model becomes linear, which is referred to as semiparametric calibration. Joint coordinates and joint offset are separated by expanding the trigonometric function into a sum of trigonometric functions. The more non-linear the original kinematic parameters are, the more linear factors are needed (model 24 → 36, model 42 → 186, model 54 → 366).

For model 24 (equation (9)) semiparametric calibration becomes also decoupled for one main chain. Hence expanding one of these equations and replacing the 24 kinematic parameters \( (p_{1-3,1-8}) \) by 36 linear factors \( (v_{1-3,1-12}) \) yields:

\[
\begin{align*}
&v_{i,1} + v_{i,2} \cdot x + v_{i,3} \cdot y + v_{i,4} \cdot z + v_{i,5} \cdot \cos \alpha_i \\
&+ v_{i,6} \cdot \sin \alpha_i + v_{i,7} \cdot x \cdot \cos \alpha_i + v_{i,8} \cdot y \cdot \cos \alpha_i \\
&+ v_{i,9} \cdot z \cdot \cos \alpha_i + v_{i,10} \cdot x \cdot \sin \alpha_i \\
&+ v_{i,11} \cdot y \cdot \sin \alpha_i + v_{i,12} \cdot z \cdot \sin \alpha_i = 0
\end{align*}
\]

\[i = 1 \ldots 3 \quad (12)\]

with

\[
\begin{align*}
&v_{i,1} = p_{i,1-3} \cdot p_{i,1-3}^T + p_{i,6-7}^T + p_{i,6-7} - p_{i,8}^2; \\
&v_{i,2} = -2(p_{i,1} \cdot \cos p_{i,4} - p_{i,2} \cdot \cos p_{i,5} \cdot \sin p_{i,4} + p_{i,3} \cdot \sin p_{i,5} \cdot \sin p_{i,4}); \\
&v_{i,3} = -2(p_{i,1} \cdot \sin p_{i,4} + p_{i,2} \cdot \cos p_{i,5} \cdot \cos p_{i,4} - p_{i,3} \cdot \cos p_{i,5} \cdot \sin p_{i,4}); \\
&v_{i,4} = -2(p_{i,5} \cdot \cos p_{i,5} + p_{i,2} \cdot \sin p_{i,5}); \\
&v_{i,5} = 2(p_{i,1} \cdot p_{i,6} + p_{i,2} \cdot p_{i,7}); \\
&v_{i,6} = 2(p_{i,1} \cdot p_{i,6} - p_{i,1} \cdot p_{i,7}); \\
&v_{i,7} = -2(p_{i,5} \cdot \cos p_{i,4} - p_{i,7} \cdot \cos p_{i,5} \cdot \sin p_{i,4}); \\
&v_{i,8} = -2(p_{i,5} \cdot \sin p_{i,4} + p_{i,7} \cdot \cos p_{i,5} \cdot \cos p_{i,4}); \\
&v_{i,9} = -2p_{i,7} \cdot \sin p_{i,5}; \\
&v_{i,10} = 2(p_{i,7} \cdot \cos p_{i,4} + p_{i,6} \cdot \cos p_{i,4} \cdot \sin p_{i,4}); \\
&v_{i,11} = 2(p_{i,7} \cdot \sin p_{i,4} - p_{i,6} \cdot \cos p_{i,5} \cdot \cos p_{i,4}); \\
&v_{i,12} = -2p_{i,6} \cdot \sin p_{i,5};
\end{align*}
\]

\[i = 1 \ldots 3 \quad (13)\]

Equation (13) shows the linear factor as a function of the kinematic parameters (Table II: \( p_{i,1} = D_{i,x} \), \( p_{i,2} = D_{i,y} \), \( p_{i,3} = D_{i,z} \), \( p_{i,4} = b_i \), etc.). However, since they are assumed to be independent, these geometric constraints are dropped, hence the name “semiparametric” calibration. From now on it is important to work consequently with the 36 linear factors. The direct and inverse problem for instance have to be solved directly from the semiparametric model (equation (12)).

By substituting the measurement points into the equation (12) the residuals of one main chain result in:

\[
\begin{align*}
&v_1 + v_2 \cdot \tilde{x}_j + v_3 \cdot \tilde{y}_j + v_4 \cdot \tilde{z}_j + v_5 \cdot \cos \tilde{\alpha}_j \\
&+ v_6 \cdot \sin \tilde{\alpha}_j + v_7 \cdot \cos \tilde{\alpha}_j + v_8 \cdot \tilde{y}_j \cdot \cos \tilde{\alpha}_j \\
&+ v_9 \cdot \tilde{z}_j \cdot \cos \tilde{\alpha}_j + v_{10} \cdot \tilde{x}_j \cdot \sin \tilde{\alpha}_j \\
&+ v_{11} \cdot \tilde{y}_j \cdot \sin \tilde{\alpha}_j + v_{12} \cdot \tilde{z}_j \cdot \sin \tilde{\alpha}_j = p_i \\
&j = 1 \ldots N \quad (14)
\end{align*}
\]

Again the square of the residuals \( (p_i) \) is used as merit function. The same set of 74 measurement points as for implicit calibration is used for identification. The resulting \( 12 \times 74 \) matrix was inverted by means of singular values decomposition. Within 0.5 seconds the linear factors \( (v_{1-3,1-12}) \) of all three main chains were identified leading to the improvement presented in Table VI.

From Table VI and Table III it can be seen that
semiparametric calibration works better than implicit calibration being 174-times faster. However, semiparametric calibration has the disadvantage of the linear factors not having a physical meaning anymore. This makes it impossible to replace mechanical parts which are out of tolerances. Such quality control is only possible by implicit calibration, which is the reason for proposing this method as standard method for calibration of parallel robots.

6. IMPLEMENTATION
The implementation step deals with the question of how to solve the direct and inverse problem of the calibration model. This is a very difficult task if trying to find the solution by reducing the non-linear system of equations to a univariate polynomial since simplifications (e.g. intersecting axes) which allow the reduction of the nominal model into a polynomial, may not be valid any more. Simple polynomial solutions can be found for model 24 whereas for model 54 this is not possible any more.8

However, numerical algorithms such as Newton-Raphson work generally well for such kind of problems. In order to reduce the calculation time solutions must be found which are based on solutions of low order polynomials.

Due to the splitting of model 54 (equation (5)) into two sets of equations (G1 and G2) a simple algorithm to solve the direct problem (Figure 13) could be found. It is polynomial based and faster than the Newton-Raphson algorithm.

<table>
<thead>
<tr>
<th>Table IV. Identified parameters of model 54 (equation (5))</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 parameters influencing the position</td>
</tr>
<tr>
<td>main chain</td>
</tr>
<tr>
<td>unit</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>30 parameters affecting orientation &amp; position</th>
</tr>
</thead>
<tbody>
<tr>
<td>main chain</td>
</tr>
<tr>
<td>unit</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Fig. 12. Orientation error of the Delta end-effector before (dashed line) and after (solid line) calibration of model 54 (equation (5)).
The direct problem of model 54 is that of calculating no, one ore several sets of world coordinates \((P, R)\) for a given set of joint angles \((Q)\). As an initial guess \((R^*)\) the orientation matrix can be set equal to the identity matrix since the end-effector is almost parallel to the base. By substituting this guess \((R^*)\) and the joint angles \((Q)\) into the first set of equations \(G1\) an estimation of the end-effector's position \((P)\) can be calculated based on a second order univariate polynomial. This estimation as well as the joint angle set is forwarded to the second set of equations \(G2\). The rotation matrix can be linearized in \(G2\) since the deviations in the parallelism are small. By doing so \(G2\) can be linearly solved for the orientation \((R)\). This procedure can be continued iteratively. It remains an error due to the linearization of the rotation matrix. However this error is small and the algorithm converging very fast.

Table VII shows a comparison of the calculation time (Motorola 68040 with co-processor) with an accelerated Newton-Raphson algorithm (no updating of the Jacobian, no precalculation of the position with the nominal model, stopping after five steps). The iteration of the cascaded iterative algorithm is stopped after one and a half steps.

The third column of Table VII gives the remaining error in the calculation of the end-effector's pose as compared to a Newton-Raphson solution of machine precision. The inverse problem of model 54 is not easy to solve since it is coupled by pairs in the joint coordinates and all six equations are coupled in the orientation angles. However, it can be solved similarly to the direct problem (Figure 13) leading to a calculation time of 1.9 seconds.

### 7. CONCLUSIONS

In this paper the kinematic calibration of the Delta robot was reported. Two parametric models were established. Model 24 takes only position errors of the end-effector into account, whereas in the model 54 models deviations from the parallelism between the base and the end-effector are also considered.

A measurement set-up was built with a measuring machine and linear probes capable to measure the full pose of the end-effector with respect to the base.

Based on the same set of 74 measurement points the parameters of model 24 and model 54 were identified. The parameter estimation problem was defined using directly the implicit closure equations, which is referred to as implicit calibration. This method enables to solve the direct and inverse problem during identification. Improvement in position of a factor of 12.3 was reached by the identified model 24. Prediction of the orientation could be improved by a factor of 3.4 for the identified model 54.

A second linear calibration method referred to as semiparametric calibration was tested. Starting by expansion of the closure equations and replacing all coefficients in front of the joint and world coordinates by linear factors leads to the semiparametric model. Its linear factors can be identified linearly which is very advantageous, compared to non-linear estimation: no initial guess is needed, no Jacobian is needed, there is no problem with multiple minima since the solution is unique, it is very fast since there is no need for iteration. And last but not least, improvement in the position is with a factor of 15.2 higher than the one of implicit calibration. However, semiparametric calibration allows no quality control of the mechanical parts of the robot and further tests will be necessary to check its reliability. Hence, implicit calibration is proposed as standard calibration method for parallel robots.

In a last section a new algorithm was presented allowing to solve the direct as well as the inverse problem of model 54 faster than with the Newton-Raphson algorithm.

The main contribution of this paper is the experimental verification of the proposed theoretical tool. It was

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**Table IV.** Position error of the end-effector before and after calibration of the semiparametric model (equation (12)) which is based on model 24

<table>
<thead>
<tr>
<th>Position error [µm]</th>
<th>(\Delta x)</th>
<th>(\Delta y)</th>
<th>(\Delta z)</th>
<th>([\Delta x, \Delta y, \Delta z])</th>
</tr>
</thead>
<tbody>
<tr>
<td>before calibration</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>260</td>
<td>350</td>
<td>-230</td>
<td>550</td>
</tr>
<tr>
<td>deviation</td>
<td>220</td>
<td>99</td>
<td>190</td>
<td>180</td>
</tr>
<tr>
<td>after calibration</td>
<td>-0.72</td>
<td>-0.36</td>
<td>0.01</td>
<td>36</td>
</tr>
<tr>
<td>deviation</td>
<td>31</td>
<td>18</td>
<td>24</td>
<td>23</td>
</tr>
</tbody>
</table>

**Table VII.** Calculation time of a Newton-Raphson algorithm and the cascaded iterative algorithm (Figure 13)

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Direct solution [s]</th>
<th>Remaining pose error [µm], [arcseconds]</th>
</tr>
</thead>
<tbody>
<tr>
<td>accelerated Newton-Raphson</td>
<td>2.7</td>
<td>([0.02, 0.3])</td>
</tr>
<tr>
<td>Cascaded iterative (Figure 13)</td>
<td>2.0</td>
<td>([0.1, 1.3])</td>
</tr>
</tbody>
</table>

![Fig. 13. Cascaded iterative algorithm to solve the direct problem of model 54 (equation (5)).](image-url)
shown that improved accuracy of the parallel Delta robot by means of calibration is possible.

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References