Robotics & Automation

Lecture 12

Closed Kinematic Chain and Parallel Mechanisms

John T. Wen

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Examples of closed kinematic chains

- Constrained robot, e.g., robot in contact with environment (polishing, assembly, etc.).
- Multiple interacting robots, e.g., multi-finger grasp, multiple cooperative robots.
- Parallel mechanism, e.g., 4-bar linkage, slider-crank, Stewart-Gough Platform, Delta robot.
Kinematics

Forward kinematics and inverse kinematics are the same as before: given joint positions, find the task frame, and vice versa – *subject to the closed chain constraints*.

Usually: Serial mechanism has complicated geometry, inverse kinematics is more difficult than the forward kinematics. Parallel mechanisms usually are based on imposing kinematic constraints to simple serial mechanisms. Therefore, inverse kinematics is usually easy. The forward kinematics needs to take into account of constraints, and is more difficult to solve.
Constrained mechanisms

Consider a serial kinematic chain with $n$ joints ($\theta \in \mathbb{R}^n$). Suppose the chain is constrained:

$$\phi(\theta) = 0, \quad \phi : \mathbb{R}^n \rightarrow \mathbb{R}^k.$$

Then the mechanism has $n - k$ (unconstrained) DOF.

Example:
- 3-DOF Planar arm with tip constrained to move along a line (2 dof)
- 4-bar linkage (1 dof)
Greubler’s Formula

Consider a planar mechanism. Let $N =$ # of links, $g =$ # of joints, $f_i =$ # of DOF of $i$th joint.

DOF: $3N$

Constraints: $3g - \sum_{i=1}^{g} f_i$.

Net # of DOF: $3(N - g) + \sum_{i=1}^{g} f_i$

for spatial mechanisms (bodies are free to translate and rotate in Cartesian space), we have

Net # of DOF: $6(N - g) + \sum_{i=1}^{g} f_i$.

Example: planar platform with three legs.
Kinematics: 4-bar linkage Example

rotation: $R = \exp(\hat{z}\theta_1)\exp(\hat{z}\theta_3) = \exp(\hat{z}\theta_2)\exp(\hat{z}\theta_4)$ (1 constraint)

position: $p_{14} = \exp(\hat{z}\theta_1)(p_{13} + \exp(\hat{z}\theta_3)p_{34}) = p_{12} + \exp(\hat{z}\theta_2)p_{24}$ (2 constraints)

Inverse kinematics: given $R$ (equiv. $\theta_E = \theta_1 + \theta_3 = \theta_2 + \theta_4$), find $(\theta_1, \theta_2, \theta_3, \theta_4)$.

Forward kinematics: Suppose $\theta_1$ is the active joint. Given $\theta_1$, find $\theta_E$.

Typically, inverse kinematics is easy (since each leg is simple), but forward kinematics hard (solving the constraint equations).
Slider Crank

Structure equation (kinematics + constraints):

\[ R_{03} = R_{01}R_{12}R_{23} = e^{(\theta_1 + \theta_2 + \theta_3)\hat{z}} \]

\[ p_{0T} = R_{01}p_{12} + R_{01}R_{12}p_{23} + R_{01}R_{12}R_{23}p_{3T} \]

\[ R_{03} = I \]

\[ y^T p_{0T} = 0. \]