Robotics & Automation

Lecture 15

Multiple robot arms, multi-finger grasps

John T. Wen

March 19, 2007
Last Time

- Forward and inverse kinematics of parallel robots.
- Jacobian and singularities (unstable and unmanipulable) for parallel robots.

Today:

- Multiple arm holding a common payload
- Multi-finger grasp
When $m$ robots rigidly hold an object, there are kinematic loops but no passive joints! In that case, the joint angles must satisfy the kinematic constraints:

$$v_i = J_i \dot{\theta}_i = A_i v_c,$$

where

$$A_i = \begin{bmatrix} I & 0 \\ \vec{p}_{ic} \times & I \end{bmatrix}.$$

Stack up the variables, we have

$$J \dot{\theta} = A v_c$$

where $A$ is a $6m \times 6$ matrix. Let $\tilde{A}$ be the annihilator of $A$. Constraint on $\dot{\theta}$ is:

$$\tilde{A} J \dot{\theta} = 0.$$
Kinematics of Multi-Finger Grasp

When $m$ fingers hold an object, there may be motion at the finger contact. The kinematics then becomes:

$$A_i v_c = v_i = J_i \dot{\theta}_i + H_i W_i$$

where $W_i$ is the (passive) contact joint velocity, $H_i$ the axes of motion. In stacked notation:

$$A v_c = J \dot{\theta} + H W.$$ 

Let $\tilde{H}$ be the annihilating matrix of $H$, then

$$\tilde{H} A v_c = \tilde{H} J \dot{\theta}.$$

$G$ is called the grasp map and $J_h$ the hand Jacobian.
Grasp Stability and Manipulability

\[ \mathcal{N}(G^T) = \{ \text{self motion of object when fingers are locked} \} \]

The grasp is stable if \( \mathcal{N}(G^T) = \{0\} \).

The grasp is manipulable if \( \mathcal{R}(G^T) \subset \mathcal{R}(J_h) \).

Stable grasp is the same concept as stable parallel mechanism:

\[ y \in \mathcal{N}(\tilde{H}A) \iff x \in \mathcal{N}(\tilde{A}H), Hx = Ay. \]
Common Contact Models

- Point contact without friction (rotation + tangential translation):

\[
H = \begin{bmatrix}
I & 0 \\
0 & \vec{h}_{t1}, \vec{h}_{t2}
\end{bmatrix}, \\
W = \begin{bmatrix}
\vec{\omega} \\
\nu_{t1} \\
\nu_{t2}
\end{bmatrix}.
\]

- Point contact with friction (rotation only):

\[
H = \begin{bmatrix}
I \\
0
\end{bmatrix}, \\
W = \vec{\omega}.
\]

- Soft finger (no rotation about surface normal):

\[
H = \begin{bmatrix}
\vec{h}_{t1} & \vec{h}_{t2} \\
0 & 0
\end{bmatrix}, \\
W = \begin{bmatrix}
\omega_{t1} \\
\omega_{t2}
\end{bmatrix}.
\]
- Hinge (rotation about a line):
  \[
  H = \begin{bmatrix} \vec{h} \\ 0 \end{bmatrix}, \quad W = \dot{\theta}.
  \]

- Surface sliding (surface translation, no rotation):
  \[
  H = \begin{bmatrix} 0 & 0 \\ \vec{h}_1 & \vec{h}_2 \end{bmatrix}, \quad W = \begin{bmatrix} v_{t1} \\ v_{t2} \end{bmatrix}.
  \]

- Line sliding (translation along line, no rotation):
  \[
  H = \begin{bmatrix} 0 \\ \vec{h} \end{bmatrix}, \quad W = v.
  \]
Dual Perspective

Force in task frame: \( f_T = A^T f \).

Since fingers are free to move along the range Of \( H \) (linear combination of columns of \( H \)), the corresponding spatial force (torque/force) is zero:

\[
H^T f = 0, \quad f = [f_1^T, \ldots, f_m^T]^T.
\]

This means that \( f = \tilde{H}^T \eta \) (where \( \tilde{H}H = 0 \)).

Note that

\[
f_T = A^T \tilde{H}^T \eta.
\]

Stable grasp condition, \( \mathcal{N}(G^T) = \{0\} \), can also be written as \( R(G) = \mathbb{R}^p \), which means finger forces can together resist any applied task force.

- Point contact without friction:

\[
\tilde{H} = \begin{bmatrix} 0 & \tilde{h}_n \end{bmatrix}, \quad \eta = f_n \text{ (normal force)}.
\]
• Point contact with friction (rotation only):

\[
\tilde{H} = \begin{bmatrix} 0 & I \\ \end{bmatrix}, \quad \eta = f \text{ (contact force)}.
\]

Friction cone (no-slide) condition: \( \| \vec{f}_t \| \leq \mu \| \vec{f}_n \|, \vec{f}_n = \vec{f} \cdot \vec{h}_n, \vec{f}_t = \vec{f} - \vec{f}_n, \mu = \text{static coefficient of friction} \).

• Soft finger (no rotation about surface normal):

\[
\tilde{H} = \begin{bmatrix} \vec{h}_n \cdot & 0 \\ 0 & I \\ \end{bmatrix}, \quad \eta = \begin{bmatrix} \tau_n \\ \vec{f} \end{bmatrix}.
\]

Similarly for others.
A finger can push but may lose contact if it pulls. This motivates the following definition: Suppose all finger contacts are frictionless (e.g., holding a soap), the grasp map \( G = [G_1 : \ldots : G_m] \in \mathbb{R}^{p \times m} \), \( p = \# \) of task DOF and \( m = \# \) of fingers, is a force closure grasp if \( \{G_i\} \) positively span \( \mathbb{R}^p \), which means every \( \mathbb{R}^p \) vector can be written as a linear combination of \( \{G_i\} \) with positive coefficients.

Equivalent conditions:

- Convex hull of \( \{G_i\} \) \( (\sum_{i=1}^m \lambda_i G_i, \sum_i \lambda_i = 1, \lambda_i \geq 0) \) contains a neighborhood of the origin.

For frictional contacts, replace \( G_i \)’s by the boundary of friction cone.