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I. INTRODUCTION

Finding minimum-time planning strategies for robot manipulators, given actuator constraints, has been a long-standing concern in the robotics literature. This interest is largely motivated by the obvious relationship between execution time of specific tasks and productivity. Because of the nonlinear multi-input dynamics of robot manipulators, however, finding true minimum-time solutions is difficult. In early methods, various assumptions or simplifications on the manipulator dynamics were used to obtain near-minimum time solutions. Kahn and Roth [4] used linearization techniques and studied the application of linear optimal control theory. Purely kinematic approaches were proposed by Lin, Chang, and Luh [5] while Schemman and Roth [11], Wen and Descrochers [16], and Singh and Leu [15] used other kinds of simplifying assumptions. In particular, the problem is considerably simplified if the robot arm is assumed to be statically balanced [6]; such an assumption, however, precludes the robot from manipulating various loads of weights and sizes similar to its own, as the human arm routinely does.

The true minimum-time solution along a prescribed path was derived by Bobrow, Dubowsky, and Gibson, [1], [2] and Shin and McKay, [13], [14], based on the possibility of parameterizing the path with a single scalar variable. Their methods consider the full nonlinear manipulator dynamics and the torque constraints, and thus provide true minimum-time solutions. Pfeiffer and Johann [7], [8] extended this method to the two-link robot.

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The authors are with the Nonlinear Systems Laboratory, Massachusetts Institute of Technology, Cambridge, MA 02139.

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press the parameterized dynamic equations slightly differently from
the former works, while using in essence the same procedure as in
Shin and McKay, [13], [14]. These algorithms, however, require a
search over the whole range of the scalar parameter and that of its
time derivative, in order to find the maximum possible manipulator
tip velocity limit for each value of the parameter.

In this communication, new characteristics of the time-optimal
planning problem for robot manipulators are identified and applied to
the existing prescribed-path algorithms. A modification is proposed,
which greatly simplifies the original procedures and provides a new
interpretation of the time-optimal problem. Characteristic switch-
ing points are defined, which are uniquely determined given the
manipulator dynamics, the actuator bounds, and the prescribed path.
Necessary conditions describing the characteristic switching points
are derived as functions of the scalar parameter defining the position
along the path. Based on the characteristic switching points, limit
curves are drawn in the phase plane, in contrast with the maximum
velocity curves of existing methods. The limit curves set sharper ad-
missible regions for the phase-plane trajectory, and shape the solution
for the time-optimal trajectory accordingly.

After a review, in Section II, of the methods of Bobrov et al. [1],
[2] and Shin and McKay, [13], [14], the new minimum-time algo-
rithm is described in Section III. A numerical example, in Section IV.
demonstrates the potential for computational improvements of
several orders of magnitude over existing methods, making quasi-
real-time trajectory generation feasible. Section V offers brief con-
cluding remarks.

II. EXISTING SOLUTIONS TO THE OPTIMAL
PLANNING PROBLEM

In the absence of friction or other disturbances, the dynamics of
a rigid manipulator can be written as

$$H(q)\ddot{q} + \dot{q}^T Q(q)\dot{q} + g(q) = \tau$$  \hspace{1cm} (1)

where $q$ is the $n \times 1$ vector of joint displacements, $\tau$ is the $n \times 1$
vector of applied joint torques (or forces), $H(q)$ is the $n \times n$
symmetric positive-definite manipulator inertial matrix, $\dot{q}^T Q(q)\dot{q}$ is the $n \times 1$
vector of centripetal and Coriolis torques (with $Q(q)$ an $n \times n \times n$
array), and $g(q)$ is the $n \times 1$ vector of gravitational torques. When the
path for the manipulator tip to follow is specified, the vector $q$ of joint
displacements can be written as a function of a single parameter $s$,
either in task space [1], [2] or in joint space [13], [14]. Therefore,
the manipulator dynamics can be expressed as $n$ equations in the
parameter $s$. Namely, in task space, the position and orientation of
the end-effector can be represented by a $6 \times 1$ vector $p$ as

$$p = \begin{pmatrix} x \end{pmatrix} = \begin{pmatrix} x \end{pmatrix} = \begin{pmatrix} x, y, z, a_1, a_2, a_3 \end{pmatrix}$$  \hspace{1cm} (2)

where $x$ is the end-effector position vector, and $a$ is composed of Euler
angles representing the orientation of the end-effector with respect to a
fixed frame. The vector $\dot{p}$ is a known smooth function of the
displacement $s$ along the path, and can be expressed in terms of the
joint variables $q$ as

$$\dot{p} = \dot{r}_q p_0 \ddot{s}$$  \hspace{1cm} (3)

where $\dot{r}_q$ defines the direct kinematics of the manipulator. Differenti-
ating (3) with respect to time twice, and solving for $\ddot{q}$ and $\dddot{q}$,
yields

$$\dddot{q} = r_{\ddot{q}} \dddot{s}$$  \hspace{1cm} (4)

$$\dddot{q} = r_{\ddot{q}} \dddot{s} + r_{\dddot{q} \ddot{s}} p_0 \dot{s}^2$$  \hspace{1cm} (5)

where the subscripts $s$ and $\dot{q}$ denote derivatives with respect to the
scalar $s$ and the vector $\dot{q}$, with $r_{\ddot{q}}$ being the manipulator Jacobian
matrix and $r_{\dddot{q} \ddot{s}}$ the Hessian of the vector function $r$. Similarly, in
joint space, the path is given as a curve smoothly parameterized by the
single scalar $s$ as

$$q = f(s).$$  \hspace{1cm} (6)

Differentiating (6) with respect to time twice, the joint velocity and
acceleration can be written as

$$\dot{q} = f_\dot{s} \dddot{s}$$  \hspace{1cm} (7)

$$\dddot{q} = f_\ddot{s} \dddot{s} + f_\dddot{s} \dddot{s}^2$$  \hspace{1cm} (8)

Using (4) and (5), or (7) and (8), with (1), we get $n$ equations of
motion parameterized by the single parameter $s$. In task space, these
are written

$$a_T(s)\ddot{s} + b_T(s)\dddot{s}^2 + c_T(s) = \tau$$  \hspace{1cm} (9)

where the $n \times 1$ vectors $a_T(s)$, $b_T(s)$, and $c_T(s)$, are defined as
(with the subscript $TS$ denoting task-space quantities)

$$a_T(s) = Hq^{-1} p_0$$  \hspace{1cm} (10)

$$b_T(s) = Hq^{-1} \{ p_0s - (r_{\ddot{q}} \ddot{s})^T r_{\dddot{q} \ddot{s}} \dot{q} \}$$

$$+ (r_{\dddot{q} \ddot{q}} \ddot{s})^T Qq^{-1} p_0$$  \hspace{1cm} (11)

$$c_T(s) = g(s).$$  \hspace{1cm} (12)

Similarly, in joint space, we obtain (with the subscript $JS$ denoting
joint-space quantities)

$$a_J(s)\ddot{s} + b_J(s)\dddot{s}^2 + c_J(s) = \tau$$  \hspace{1cm} (13)

where the $n \times 1$ vectors, $a_J(s)$, $b_J(s)$, and $c_J(s)$, are defined as

$$a_J(s) = Hf \ddot{s}$$  \hspace{1cm} (14)

$$b_J(s) = Hf \dddot{s} + f_\ddot{s} Q \ddot{s}$$  \hspace{1cm} (15)

$$c_J(s) = g(s).$$  \hspace{1cm} (16)

Note that (9) and (13) have the same form, and that only the coef-
icient vectors $a$, $b$, and $c$ are different. We shall write both equations in
the form

$$a_i(s)\ddot{s} + b_i(s)\dddot{s}^2 + c_i(s) = \tau_i, \hspace{1cm} i = 1, 2, \cdots, n.$$  \hspace{1cm} (17)

The joint actuator torques (or forces) $\tau_i$ are assumed to be bounded
by constants

$$\tau_i^{\min} \leq \tau_i \leq \tau_i^{\max}, \hspace{1cm} i = 1, 2, \cdots, n.$$  \hspace{1cm} (18)

Viscous damping effects, and especially the back EMF of the joint
actuator motors, can also be included in the dynamic equations (1); such
case shall be detailed later in Section III-D.

Given the actuator bounds (18), the maximum and minimum possible
values of the parameter $s$ can be determined as functions of $s$
and $\dot{s}$ as

$$s_i^{\min} \leq a_i(s)\ddot{s} + b_i(s)\dddot{s}^2 + c_i(s) \leq s_i^{\max},$$

for all $i = 1, 2, \cdots, n$ (19)

which can be written formally as $n$ sets of constraints on $\ddot{s}$

$$a_i(s, \dot{s}) \ddot{s} \leq \ddot{s} \leq b_i(s, \dot{s})$$  \hspace{1cm} (20)

where

$$a_i = (r_{\ddot{q}} \ddot{s} - b_i s^2 - c_i)/a_i$$  \hspace{1cm} (21)

$$\beta_i = (r_{\dddot{q} \ddot{q}} \ddot{s}^2 - c_i)/a_i$$  \hspace{1cm} (22)

and the values of $\tau_i^{\min}$ and $\tau_i^{\max}$ depend on the sign of the "inertia" term

$$a_i$$

if $a_i > 0$, \hspace{0.5cm} $\tau_i^{\min} = \tau_i^{\max}$ and $\tau_i^{\max} = \tau_i^{\max}$

if $a_i < 0$, \hspace{0.5cm} $\tau_i^{\min} = \tau_i^{\min}$ and $\tau_i^{\max} = \tau_i^{\max}$
III. IMPROVING THE EFFICIENCY OF THE OPTIMAL PLANNING ALGORITHM

In this section, we show that the points, which we shall refer to as characteristic switching points, where the phase plane trajectory just meets the maximum velocity curve without violating the actuator constraints, can be exhaustively classified into three possible types. We call these types the zero-inertia point, the discontinuity point, and the tangent point. We describe how the characteristic switching points can be directly obtained without computing the maximum velocity curve explicitly, and how this property can be exploited to simplify the derivation of the time-optimal solution.

Section III-A finds the switching points where the maximum velocity curve is continuous but not differentiable (zero-inertia points), which correspond to having one of the $a_i$ change signs. Section III-B covers the case where the maximum velocity curve is discontinuous (discontinuity points). Section III-C, which represents the main result of this communication, derives a simple procedure to find the switching points where the maximum velocity curve is continuous and differentiable (tangent points). Inclusion of viscous friction effects and state-dependent actuator bounds is detailed in Section III-D. The resulting algorithm is summarized in Section III-E.

A. Case 1: The Zero-Inertia Point

If, in (17), $a_i(s) = 0$ for some $i$, then the corresponding terms $\alpha_i$ and $\beta_i$ of (21) and (22) cannot be defined. In this case the acceleration $\dot{s}$ at the maximum velocity $s_{max}$ is not uniquely determined. The time-optimal phase plane trajectory may include this singular point, which is, therefore, a candidate characteristic switching point, as noticed in [7]. We call this case the zero-inertia point, since $a_i(s)$ represents an inertia-like term in the parameterized dynamic equation. Zero-inertia points can be found directly from the expression of $a_i(s)$.

B. Case 2: The Discontinuity Point

In this subsection and the next, we shall assume that none of the $a_i(s)$ is zero in the vicinity of the characteristic switching point considered (this case having been treated in the previous subsection). We can further assume, without loss of generality, that the first derivative of the parameterized path function, namely $p_i$ of [1] and [2], or $f_i$ of [13] and [14], is continuous. Indeed, if this is not the case at a particular point, then at this point the velocity along the path is necessarily zero (since, physically, the velocity vector cannot be discontinuous), so that the task can be partitioned into two independent optimal control problems.

The second derivative $p_{ii}$ (or $f_{ii}$), however, may be discontinuous. Assume that for a given value of $s$ the maximum possible value of $\dot{s}$, $s_{max}$, is attained at an extreme point $\dot{s}$ of $s_{max}$ (or $f_{ii}$) at $s$. The maximum value limit $\dot{s} = s_{max}$ is satisfied by any extreme point $\dot{s}$ of $s_{max}$, but the maximum value $s_{max}$ is discontinuous in that vicinity. Assuming as we do that none of the $a_i(s)$ is zero, the only component which may be discontinuous is $a_i(\dot{s})$. Therefore, the maximum velocity curve is discontinuous (Fig. 2) and continuous (Fig. 2) if only if $s_{max}$ is discontinuous.

For example, if the path is parameterized in task space, and the parameter $s$ is the displacement along the path from the starting point, then the position components $x_{ai}$ of $p_{ai}$ are normal to the path, with the magnitude of $x_{ai}$ being the reciprocal of the radius of the path. In this case, the discontinuity point is where the curvature of the path changes discontinuously.
C. Case 3: The Tangent Point

Subsection III-A determined the switching points where the maximum velocity curve is continuous but not differentiable (which corresponds to having one of the \( a_i \) be zero). Subsection III-B covered the case where the maximum velocity curve is discontinuous. This subsection completes the analysis, by deriving a simple procedure to find the switching points where the maximum velocity curve is both continuous and differentiable.

The smoothness of the maximum velocity curve at the switching point implies that \( \dot{s} \) is continuous in the vicinity of that point, and thus that the phase-plane trajectory must be locally continuous and differentiable. Therefore, if the trajectory hit the maximum velocity curve other than tangentially, then it would enter the region above the maximum velocity curve (Fig. 4(a)). Hence, the trajectory must meet the maximum velocity curve tangentially (Fig. 3(a)). At any time, at least one of the actuators must be saturated [11, 2], and at the switching point there must exist another actuator which is also saturated. Specifically, assume that deceleration continues just before and after the switching point, with the \( k \)th actuator saturated, i.e.,

\[
\dot{s} = \alpha_k(s, \dot{s}) = \left( \tau_k^0 - b_k(s)\dot{s}^2 - c_k(s) \right) / a_k(s)
\]  

(24)

then at the switching point there is another saturated torque \( \tau_m (m \neq k) \). Since by using (17) and (24) the torque \( \tau_m \) can be expressed smoothly in terms of \( s \) and \( \dot{s} \), which are both continuous and differentiable at the switching point

\[
\tau_m = a_m(s) \left[ \tau_k^0 - b_k(s)\dot{s}^2 - c_k(s) \right] / a_k + b_m(s)\dot{s}^2 + c_m(s)
\]

(25)

thus \( \tau_m \) must itself be continuous and differentiable. Therefore, if its torque trajectory hit the corresponding bound other than tangentially, then \( \tau_m \) would have to violate its constraint (Fig. 4(b)). Hence the torque \( \tau_m \) must meet its constraint bound tangentially (Fig. 3(b)). Based on this analysis, a necessary condition for the point \( (s^*, \dot{s}^*) \) in the phase plane to be a tangent point can be derived as follows. Since \( \tau_m \) must meet the actuator constraint tangentially

\[
d\tau_m = 0.
\]

(26)

Since, from (25), \( \tau_m \) can be expressed as a function of only \( s \) and \( \dot{s} \), condition (26) is equivalent to

\[
\frac{d\tau_m}{ds} \dot{s} + \frac{d\tau_m}{d\dot{s}} \ddot{s} = 0.
\]

(27)

Using (24), (25), and (27), yields

\[
\phi_1(s)\dot{s}^2 + \phi_2(s) = 0
\]

(28)

where

\[
\phi_1(s) = \frac{d\phi_1}{ds} - 2a_1 \frac{b_k}{a_k}
\]

\[
\phi_2(s) = \frac{d\phi_2}{ds} - 2a_1 \frac{c_k}{a_k}
\]

(29)
and
\[ \eta_1(s) = b_m - b_k a_m / a_k \]
\[ \eta_2(s) = c_m - c_k a_m / a_k. \]  (30)

Therefore, for each value of \( s \), we can solve (28) for \( s^* \) (recalling that by definition \( s \geq 0 \)), and then check for the value \((s^*, \dot{s}^*)\) whether
\[ \tau_m = \tau_m^{\max} \quad (a_m > 0) \quad \text{or} \quad \tau_m = \tau_m^{\min} \quad (a_m < 0) \]
while the other actuator torques remain within their admissible bounds. If this is the case, then \((s^*, \dot{s}^*)\) is a possible tangent point. Note that in (29) the derivative terms \( d/ds \) need not be computed explicitly, but rather can be approximated with appropriate accuracy using, for example, a central differentiation method, since the actuator torque is assumed to be a smooth function of \( s \) and \( \dot{s} \) in the vicinity of the tangent point. Also, since, by definition, \( k \) and \( m \) cannot be equal, there are only \( n(n - 1)/2 \) possible combinations of joints for (28). These combinations can be computed in a parallel and independent fashion, so that the computation time may only increase modestly with the number of links.

D. Viscous Friction Effects and State-Dependent Actuator Bounds

When viscous friction effects (such as the back EMF of the motors) are included, the parameterized equations can be expressed as
\[ a_i(s) \ddot{s}_i + b_i(s) \ddot{s}_i^2 + c_i(s) + d_i(s) \dot{s}_i = 0, \quad i = 1, \ldots, n \]
instead of (17). More generally, we may include any smooth state-dependent actuator bounds. Assume for instance that the actuator bounds have the following form, which corresponds to a fixed-field dc motor with a bounded input voltage:
\[ \tau_i^{\max} = V_i^{\max} + K_i q_i = V_i^{\max} + K_i \sigma(s) \dot{s} \]
\[ \tau_i^{\min} = V_i^{\min} + K_i q_i = V_i^{\min} + K_i \sigma(s) \dot{s} \]
where \( V_i^{\max} \) and \( V_i^{\min} \) are (scaled) input voltage bounds, \( K_i \) is a constant coefficient, and
\[ \sigma(s) = f_{\theta}^{-1} p_1 \quad \text{in task space} \]
\[ = f_{\theta} \quad \text{in joint space}. \]
The tangentiality condition (26) is then modified as
\[ d \tau_m = d \tau_m^* \]
and the necessary condition (28) becomes accordingly
\[ \phi_1(s) \dot{s}^2 + \phi_2(s) \ddot{s}^2 + \phi_3(s) \dot{s} + \phi_4(s) = 0 \]  (31)

where
\[ \phi_1(s) = \frac{d \eta_1}{ds} - 2 \eta_1 \frac{b_k}{a_k} \]
\[ \phi_2(s) = \frac{d \eta_2}{ds} - K_m \frac{d a}{ds} + 2 \eta_1 \left( \frac{K_k \sigma}{a_k} - \frac{d_k}{a_k} \right) \]
\[ - \frac{b_k}{a_k} ( \eta_3 - K_m \sigma) \]
\[ \phi_3(s) = \frac{d \eta_3}{ds} + 2 \eta_1 \left( \frac{V_k^{\alpha}}{a_k} - \frac{c_k}{a_k} \right) \]
\[ + ( \eta_3 - K_m \sigma) \left( \frac{K_k \sigma}{a_k} - \frac{d_k}{a_k} \right) \]
\[ \phi_4(s) = ( \eta_3 - K_m \sigma) \left( \frac{V_k^{\alpha}}{a_k} - \frac{c_k}{a_k} \right). \]

In this case, as remarked in [14], an exclusion "island" may occur in the phase plane, which in our formalism simply corresponds to the fact that for each \( s \), (31) may have multiple positive solutions in \( \dot{s} \). The necessary condition developed above must hold at the boundary of the island.

E. Summary

We have described how to find all the possible points where the phase-plane trajectory can meet the maximum velocity curve (or exclusion islands) in the time-optimal solution. These three types of points are uniquely determined from the given path, the manipulator dynamics, and the actuator constraints, and are independent of the initial and final positions and velocities along the path. The conditions determining the characteristic switching points are expressed as functions of only the single parameter \( s \). This allows us to avoid the exhaustive searches of the earlier methods over the whole range of \( s \) and \( \dot{s} \). The new algorithm can be summarized as follows:

1) Using the methods explained in Sections III-A, -B, and -C, obtain all the candidate characteristic switching points by searching once over the values of \( s \). The maximum velocity curve need not be found explicitly.

2) From those points, integrate \( s = \alpha(s, \dot{s}) \) backward in time, and \( \dot{s} = \beta(s, \dot{s}) \) forward in time, until the phase-plane trajectory hits the \( s \) axis, the \( \dot{s} \) axis, or the \( s = 0 \) line, or until one of the actuator constraints is violated. If, from a candidate characteristic switching point, one cannot integrate both forward and backward without violating the actuator constraints, then such point should be discarded.
Then, the admissible region for the phase-plane trajectory is under resulting limit curves. The procedure is described graphically in Fig. 5. The maximum velocity curve is also shown, in order to help understand how the new algorithm differs from the earlier methods. Note that the shaded region in Fig. 5 is inadmissible. This means that once the phase-plane trajectory gets into the shaded region, then later in the trajectory it cannot get out of the region without hitting the maximum velocity curve other than tangentially (in other words, without violating at least one of the actuator bounds). Therefore, the admissible region is under these limit curves, and not merely under the maximum velocity curve used in the earlier methods.

3) The characteristic switching points (points c and e in Fig. 6) have been found in steps 1) and 2). In step 3), we get the rest of the switching points as follows (Fig. 6). If the final point is between 0 and $s_f$, then there is only one switching point between a and b, which is the point where the deceleration trajectory, integrated backward in time from the final point, meets the acceleration trajectory, integrated forward in time from the starting point. If the final point is between $s_f$ and $s_{f1}$ and $s_{f2}$, then there are three switching points, namely, b, c, and some point between c and d, where the deceleration trajectory from the final point hits. Similarly, if the final point is between $s_{f2}$ and $s_{f3}$, then there are five switching points, namely, b, c, d, e, and some point between e and f. By repeating the above reasoning, all switching points can be obtained. Note that this implies that the number of switching points must be odd.

4) If the condition for a tangent point is satisfied over a some finite interval, then the phase-plane trajectory coincides with the maximum velocity curve along a finite arc.

IV. NUMERICAL EXAMPLE

The efficiency of the new algorithm is demonstrated on a hypothetical planar two-link manipulator. Each link has unit mass and unit length, and the actuator torques $\tau_1$ and $\tau_2$ are limited in absolute value to 30 and 10 N·m. The parametrized equations for these two paths are

$$x = s + 0.5 \quad y = 4(s - 0.5)^2.$$

The result of the algorithm is shown in Fig. 7. Fig. 7(a) is the limit curve which is obtained by the above method. The maximum velocity curve from the earlier method is also drawn with the limit curve in Fig. 7(b), in order to show the consistency between the two methods. Fig. 7(c) represents the complete time-optimal phase plane trajectory. If $s_{max}$ is between 0 and 0.737, then there is a single switching point. If $s_{max}$ is between 0.737 and 0.944, then there are three switching points. If $s_{max}$ is between 0.944 and 1, then there are five switching points. Fig. 7(d) gives the corresponding actuator histories.

This algorithm owes its efficiency to the fact that it does not need to compute the maximum velocity curve, and that all the characteristic switching points can be obtained by searching just once over the value of $s$ using a simple method, while earlier approaches need systematic searches to find a switching point with the help of the maximum velocity curve, and should repeat them as long as there are more switching points ahead. An exact efficiency comparison between the earlier methods and this new algorithm is difficult, since, for instance, the step sizes in $s$ and $s$ which are used to get the max-
V. CONCLUDING REMARKS

In this communication, an improved solution concept to address the time-optimal path-following problem for robot manipulators is presented and a new efficient trajectory planning algorithm is proposed. Characteristic switching points are defined and are exhaustively classified into three types. A simple method is developed to find such points. Limiting curves are introduced according to the characteristic switching points, and they set a new admissible region for the phase-plane trajectory, which differs from the admissible region defined by the maximum velocity curve of the existing methods. Numerical examples show the consistency of the new method with the existing methods, while demonstrating significant improvements in computational efficiency.

Solutions of more complex time-optimal problems, such as finding a minimum-time path for given end states or taking obstacles into account, and issues of robustness to model uncertainty, are important further research topics to be addressed. The analytic tools presented here are likely to contribute to research on those complex problems as well. Sahar and Hollerbach [10] simplify the global time-optimal path search by using dynamic time scaling and joint tessellation, but their approach is still very intensive computationally. Other global time-optimal studies such as Dubowsky, Norris, and Shiller [3], Shiller [12], and Rajan [9] use the prescribed path algorithm as a major component. Besides greatly simplifying the prescribed path algorithm itself, the necessary conditions determining the characteristic switching points, developed in Section III, must be satisfied for any globally time-optimal path as well. Thus they may allow us to analytically investigate the behavior of the characteristic switching points as the path changes, and therefore provide stronger tools to address the global time-optimal problem.

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