Solution 3.3 Note that

$$x(t) = e^{A(t-t_0)}x(t_0)$$

Then

$$\begin{bmatrix} 4 \\ -2 \end{bmatrix} = e^A \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \begin{bmatrix} 5 \\ -2 \end{bmatrix} = e^A \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$e^A = \begin{bmatrix} 4 & 5 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7071 & -0.4472 \\ -0.7071 & 0.8944 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.7071 & -0.4472 \\ -0.7071 & 0.8944 \end{bmatrix}^{-1}$$

Hence

$$x(n) = e^{nA} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.7071 & -0.4472 \\ -0.7071 & 0.8944 \end{bmatrix} \begin{bmatrix} 2^n & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.7071 & -0.4472 \\ -0.7071 & 0.8944 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and

$$A = \begin{bmatrix} 0.7071 & -0.4472 \\ -0.7071 & 0.8944 \end{bmatrix} \begin{bmatrix} \ln 2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.7071 & -0.4472 \\ -0.7071 & 0.8944 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1.3862 & 0.6931 \\ -1.3862 & -0.6931 \end{bmatrix}$$

Solution 3.4 Use PBH test: (F, G) is controllable if and only if

$$\begin{pmatrix} F-\lambda I & G \end{pmatrix} = \begin{pmatrix} A-\lambda I & 0 & B \\ C & -\lambda I & 0 \end{pmatrix}$$

has full row rank for all λ. Since (A, B) is controllable, $\begin{pmatrix} A-\lambda I & B \end{pmatrix}$ has full row rank for all λ. Thus $\begin{pmatrix} F-\lambda I & G \end{pmatrix}$ has full row rank if and only if it has full row rank for $\lambda = 0$ which implies that $\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ has full row rank.

Solution 3.7 (1) Two zeros are at -2.1861 and 0.6861 with

$$\begin{bmatrix} x \\ u \end{bmatrix} = \begin{bmatrix} -0.3015 \\ 0.8098 \\ 0.1507 \\ -0.4803 \end{bmatrix}, \begin{bmatrix} -0.8739 \\ -0.1627 \\ 0.4370 \\ -0.1371 \end{bmatrix}$$

$$\begin{bmatrix} y \\ v \end{bmatrix} = \begin{bmatrix} -0.0878 \\ 0.4719 \\ -0.8560 \\ -0.1920 \end{bmatrix}, \begin{bmatrix} -0.4636 \\ -0.1726 \\ 0.8088 \\ 0.3181 \end{bmatrix}$$

(2) No transmission zero.

(3) The only zero is -2. Since -2 is the blocking zero, $v^* G(-2) = 0$ for any v.

(4) The zeros can be found by first finding a state space realization and then using MATLAB command **tzero** or **szeros**. We can also find the zeros by finding the McMillian form:

$$M(s) = \begin{bmatrix} \frac{1}{(s+1)(s-2)(s+3)} & \\ & \frac{s^2+16s+14}{s-2} \end{bmatrix}$$

so the zeros are $z_1 = -8 + 5\sqrt{2}$, $z_2 = -8 - 5\sqrt{2}$. The zero input directions for z_1 and z_2 are respectively

$$u_1 = \begin{bmatrix} 0.5774 \\ 0.8165 \end{bmatrix}, \quad u_2 = \begin{bmatrix} -0.5714 \\ 0.8165 \end{bmatrix}$$

such that $G(z_1)u_1 = 0$ and $G(z_2)u_2 = 0$.

The zero output directions for z_1 and z_2 are respectively

$$v_1 = \begin{bmatrix} 0.2358 \\ 0.9718 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -0.9927 \\ 0.1204 \end{bmatrix}$$

such that $v_1^* G(z_1) = 0$ and $v_2^* G(z_2) = 0$.