Solution 4.7 Let G have the following state space realization

$$G(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right].$$

Then the Hamiltonian matrix is

$$H = \begin{bmatrix} A + BR^{-1}D^*C & BR^{-1}B^* \\ -C^*(I + DR^{-1}D^*)C & -(A + BR^{-1}D^*C)^* \end{bmatrix}, \quad R = \gamma^2 I - D^*D.$$

The MATLAB program in Table 4.1 will calculate the distance of the eigenvalues of H from the imaginary axis. The result is shown in Figure 4.1.

So $\|G\|_\infty \approx 2$ which is the same as from hinfnorm.

Solution 4.8 The results are listed in the following table. The frequency responses are calculated using log scale, for example, $w = logspace(-1, 1, 50)$.

ξ	1	0.1	0.01	0.001
state space	1	3.576	35.36	353.6
50pts in $[0.1, 10]$	0.9852	3.5314	7.8893	8.064
100pts in $[0.1, 10]$	0.9852	3.565	14.458	15.723
200pts in $[0.1, 10]$	0.9852	3.5735	23.522	30.9735
400pts in $[0.1, 10]$	0.9852	3.5751	30.888	60.8893

This program calls hfplot

```
g11=nd2sys([1,1],[1,5,6]);
g12=nd2sys([1,0],[1,1]);
g21=nd2sys([1,0,-2],[1,7,12]);
g22=nd2sys([1,4],[1,3,2]);
sys=sbs(abv(g11,g21),
abv(g12,g22));
gupper=5;
gstep=0.01;
[ga,dis]=hfplot(sys,gupper,gstep)
plot(ga,dis)
```

This function computes the distance:

```
function [ga,dis]=hfplot(sys,gupper,gstep)
[a,b,c,d]=unpck(sys);
glow=norm(d);
[m,k]=size(d);   ga=[];dis=[];
gamma=glow+gstep;
while gamma < gupper
   r=gamma^2*eye(k)-d'*d;
   ir=inv(r);   ar=a+b*ir*d'*c;
   H=[ar,b*ir*b';-c'*(eye(m)+d*ir*d')*c,-ar'];
   eigH=eig(H);    tmp=min(abs(real(eigH)));
   dis=[dis,tmp];    ga=[ga,gamma];
   gamma=gamma+gstep;
end
```

Table 4.1: Program for Problem 4.7

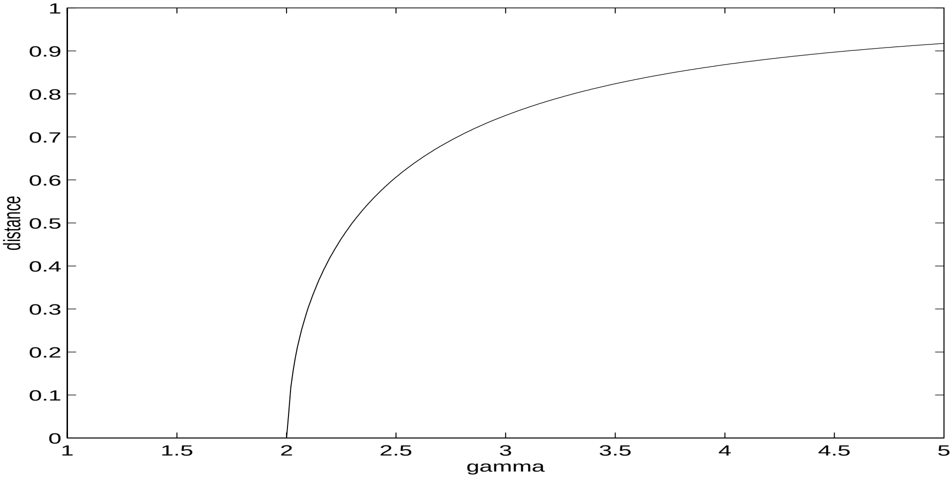

Figure 4.1: Distance vs. γ