Solution 5.1 Let the outputs of the summations be y_r, y_d and y_n respectively. Then

$$\begin{pmatrix} y_r \\ y_d \\ y_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & -H \\ G_1 & 0 & 0 \\ 0 & G_2 & 0 \end{pmatrix} \begin{pmatrix} y_r \\ y_d \\ y_n \end{pmatrix} + \begin{pmatrix} r \\ d \\ n \end{pmatrix}$$

and

$$\begin{pmatrix} y_r \\ y_d \\ y_n \end{pmatrix} = \begin{pmatrix} I & 0 & H \\ -G_1 & I & 0 \\ 0 & -G_2 & I \end{pmatrix}^{-1} \begin{pmatrix} r \\ d \\ n \end{pmatrix}$$

So the feedback system is well-posed if

$$\begin{pmatrix} I & 0 & H(\infty) \\ -G_1(\infty) & I & 0 \\ 0 & -G_2(\infty) & I \end{pmatrix} \quad \text{or} \quad I + H(\infty)G_2(\infty)G_1(\infty)$$

is nonsingular and the system is internally stable iff

$$\begin{pmatrix} I & 0 & H \\ -G_1 & I & 0 \\ 0 & -G_2 & I \end{pmatrix}^{-1} =$$

$$\begin{pmatrix} (I+HG_2G_1)^{-1} & -HG_2(I+G_1HG_2)^{-1} & -H(I+G_2G_1H)^{-1} \\ G_1(I+HG_2G_1)^{-1} & (I+G_1HG_2)^{-1}G_1 & -G_1H(I+G_2G_1H)^{-1} \\ G_2G_1(I+HG_2G_1)^{-1} & G_2(I+G_1HG_2)^{-1} & (I+G_2G_1H)^{-1} \end{pmatrix} \in \mathcal{RH}_\infty$$

If G_1 and H are both stable, then the system is internally stable iff

$$G_2(I+G_1HG_2)^{-1} \in \mathcal{RH}_\infty$$

Solution 5.7

$$G(s) = \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 1 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \end{array}\right]$$

$$F = \begin{bmatrix} 5.9675 & 0.3758 & -1.8884 & 0.1681 \\ 1.2026 & 4.4264 & -0.6588 & -0.0143 \end{bmatrix}$$

$$\begin{bmatrix} N \\ M \end{bmatrix} = \left[\begin{array}{cccc|cc} -4.9675 & -0.3758 & 1.8884 & -0.1681 & 1 & 0 \\ -1.2026 & -2.4264 & 0.6588 & 0.0143 & 0 & 1 \\ -5.9675 & -0.3758 & 1.8884 & -0.1681 & 1 & 0 \\ -1.2026 & -4.4264 & 0.6588 & -1.9857 & 0 & 1 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ \hdashline -5.9675 & -0.3758 & 1.8884 & -0.1681 & 1 & 0 \\ -1.2026 & -4.4264 & 0.6588 & 0.0143 & 0 & 1 \end{array}\right]$$

```
f=lqr(a,b,c'*c,eye(2))
N=pck(a-b*f,b,c,zeros(2,2))
M=pck(a-b*f,b,-f,eye(2))
w=logspace(-1,2,300);
Nf=frsp(N,w);
Mf=frsp(M,w);
NpM=madd(mmult(cjt(Nf),Nf), mmult(cjt(Mf), Mf));
```

Indeed this is an identity matrix for all frequency.

Solution 5.8 The state space representations of the coprime factorizations are not unique. The results are given for the specific state space realizations given below for G_1 and G_2.

$$G_1(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{s+3}{(s+1)(s-2)} \\ \frac{10}{s-2} & \frac{5}{s+3} \end{bmatrix} = \left[\begin{array}{cccc|cc} -1 & 0 & 0 & 0 & 1 & -1 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \\ 0 & 0 & 0 & -3 & 0 & 5 \\ \hline 1 & 0 & 5/3 & 0 & 0 & 0 \\ 0 & 10 & 0 & 1 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} N_1 \\ M_1 \end{bmatrix} = \left[\begin{array}{cccc|cc} -1.2126 & -3.9157 & -0.1815 & -0.3639 & 1 & -1 \\ -0.1577 & -9.4843 & -0.1880 & -0.6967 & 1 & 0 \\ 0.0548 & -3.5686 & -2.0066 & -0.3328 & 0 & 1 \\ 0.2742 & -17.8431 & -0.0330 & -4.6641 & 0 & 5 \\ \hline 1 & 0 & 1.6667 & 0 & 0 & 0 \\ 0 & 10 & 0 & 1 & 0 & 0 \\ \hdashline -0.1577 & -7.4843 & -0.1880 & -0.6967 & 1 & 0 \\ 0.0548 & -3.5686 & -0.0066 & -0.3328 & 0 & 1 \end{array}\right]$$

$$G_2(s) = \frac{s+2}{s+3} \begin{bmatrix} \frac{2(s+1)}{s(s+4)} & \frac{1}{s+1} \end{bmatrix} = \left[\begin{array}{cccc|cc} -3 & -2 & -2 & 1 & 0 & 0 \\ 0 & -4 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ \hline 1 & 2 & 2 & -1 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} N_2 \\ M_2 \end{bmatrix} = \left[\begin{array}{cccc|cc} -3 & -2 & -2 & 1 & 0 & 0 \\ -0.2672 & -4.6530 & -1.4165 & 0.3375 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.1795 & 0.3375 & 0.6139 & -1.2357 & 0 & -1 \\ \hline 1 & 2 & 2 & -1 & 0 & 0 \\ \hdashline -0.2672 & -0.6530 & -1.4165 & 0.3375 & 1 & 0 \\ -0.1795 & -0.3375 & -0.6139 & 0.2357 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} N_3 \\ M_3 \end{bmatrix} = \left[\begin{array}{ccc|ccc} -13.6163 & -12.9512 & -0.1477 & 1 & 2 & 3 \\ -8.1085 & -12.4855 & -5.6652 & 3 & 2 & 1 \\ -9.1812 & -9.3592 & -3.4532 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 3 & 4 & 0 & 0 & 0 \\ \hdashline -0.6001 & -3.0033 & -1.3638 & 1 & 0 & 0 \\ -1.7271 & -2.1197 & -0.4844 & 0 & 1 & 0 \\ -2.8540 & -1.2361 & 0.3950 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{bmatrix} N_4 \\ M_4 \end{bmatrix} = \left[\begin{array}{cc|cc} -2.8974 & -5.6922 & 1 & 2 \\ -1.0618 & -4.1854 & 2 & 1 \\ \hline 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \hdashline -0.0754 & -2.2262 & 1 & 0 \\ -0.9110 & -0.7330 & 0 & 1 \end{array}\right]$$

Solution 5.9 Take the transpose of the transfer matrix G and perform the normalized right coprime factorization. Then transpose back the coprime factors.