ECSE.6460 **Multivariable Control Systems** **Fall 2006**

MIDTERM EXAM

Due: 4:00pm, October 23, 2006 Assigned: October 16, 2006

Name: ____________________________

This is a take home exam. You have a week to complete the test. It is essential to **SHOW ALL STEPS IN YOUR WORK**. In NO circumstance is **COLLABORATION** allowed. There are four problems with a total of 100 points. Please send all questions by email to `wenj@rpi.edu`. All questions and answers will be broadcasted to all, so check your emails regularly!

Problem	Points	Score
1	10	
2	20	
3	10	
4	20	
5	20	
6	10	
7	10	
Total	100	

PLEASE ATTACH THIS SHEET TO THE TOP OF YOUR EXAM.

1. (10%) Given a minimal realization of $G(s)$, (A,B,C,D), find a state space realization of $G(s)+G^T(-s)$. Is the realization minimal?

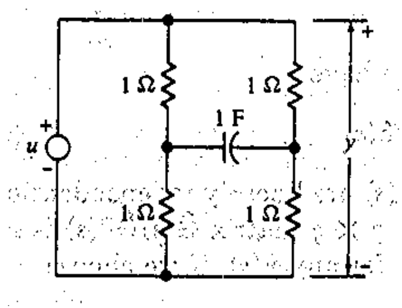
2. (20%) An LTI system $G \in \mathbb{R}^{p\times m}(s)$ is output controllable if for any $y_1 \in \mathbb{R}^p$, there exists a finite time $T > 0$ and an input $\{u(t) : t \in [0,T]\}$ that transfers the output from $y(0) = 0$ to $y(T) = y_1$.

   (a) (10%) Given that G is strictly proper and has a minimal realization given by $(A,B,C,0)$, show that G is output controllable if and only if the following matrix is of rank p:

   $$\begin{bmatrix} CB & CAB & \dots & CA^{n-1}B \end{bmatrix}.$$

   (b) (5%) Based on the previous part, does state controllability imply output controllability? If yes, prove it; if no, find a counter-example.

   (c) (5%) For the circuit below, is the system controllable? observable? output controllable?



3. (10%) Prove that (A,B) is controllable if and only if $(A+BF,B)$ is controllable for all F.

4. (20%) Given

$$G(s) = \begin{bmatrix} \frac{s}{(s+2)^2(s+1)^2} & \frac{s}{(s+2)^2} \\ -\frac{s}{(s+2)^2} & -\frac{s}{(s+2)^2} \end{bmatrix}.$$

   (a) (10%) Find all polynomial matrices in the generalized Bezout Identity:

   $$\begin{bmatrix} U_r & V_r \\ D_\ell & -N_\ell \end{bmatrix} \begin{bmatrix} N_r & V_\ell \\ D_r & -U_\ell \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}.$$

   (b) (10%) Use the right coprime MFD to find a minimal state space realization.

5. (20%) A system satisfies the Pole Interlace Property (PIP) if there are an even number of poles between consecutive **real** RHP zeros (including zeros at infinity).

   Prove that if an SISO system $g(s)$ does not satisfy PIP then it cannot be stabilized by a *stable* compensator $k(s)$.

   (a) (15%) Do the proof in the following steps:

      i. Let $k(s)$ be a stable stabilizing controller for $g(s)$. Let $\frac{n_g(s)}{d_g(s)}$ be a stable coprime factorization. Write down a Bezout Identity involving (n_g, d_g) and k.

ii. Let z_1 and z_2 be any two consecutive, real zeros of $g(s)$ ($z_2 > z_1$). From the Bezout Identity, show that $d_g(z_1)d_g(z_2) > 0$.

iii. Let $\ell =$ number of real zeros of $d_g(s)$ between z_1 and z_2. Show that

$$\operatorname{sgn}(d_g(z_2)) = (-1)^{\ell}\operatorname{sgn}(d_g(z_1)).$$

iv. Conclude the stated result from the above.

(b) (5%) Let $g(s) = \frac{s}{s^2-1}$. Show that $g(s)$ does not satisfy PIP. Verify that $g(s)$ cannot be stabilize by a stabilizing controller by finding *all* stabilizing controllers for $g(s)$.

6. (10%) Given

$$G(s) = \frac{0.8}{(s+1)(s+2)}\begin{bmatrix} s-1 & s \\ -6 & s-2 \end{bmatrix}.$$

Find the range of k such that the unity feedback system of $kG(s)$ is internally stable.

7. (10%) Consider

$$G(s) = \begin{bmatrix} \frac{s+1}{s-1} & \frac{s+4}{(s+1)^2(s-2)} \\ \frac{10s}{s-2} & \frac{5}{(s+3)(s+1)} \end{bmatrix}.$$

(a) (5%) Decompose $G(s)$ into components in H_2 and $H_2^{\perp}$. Calculate the H_2-norm of the H_2 component and $H_2^{\perp}$-norm of the $H_2^{\perp}$ component.

(b) (5%) Decompose $G(s)$ into components in H_∞ and $H_\infty^{\perp}$. Calculate the H_∞-norm of the H_∞ component and $H_\infty^{\perp}$-norm of the $H_\infty^{\perp}$ component.