DESIGN OF ADAPTIVE OPTICS BASED SYSTEMS BY USING MEMS DEFORMABLE MIRROR MODELS

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MEMS deformable mirror technology has recently evolved from a specialized wavefront control device to a low cost and high performance product. As adaptive optics technology advances, MEMS deformable mirror technology is expected to continue to grow in importance. This paper discusses the design of adaptive optics systems using MEMS deformable mirrors. The design of an adaptive optics system using MEMS deformable mirrors is presented, and the results of the design process are discussed.

Keywords: Wavefront compensation; Adaptive optics; MEMS deformable mirror; Multidisciplinary design optimization; Optical design

INTRODUCTION

Traditionally, optical systems have been composed of primarily static elements, using moving or actuated components for essential functionality only (e.g., focusing, scanning, delaying, etc.). As such, achieving high optical performance has relied on complex and expensive optical designs consisting of a large number of optical elements combined with accurate manufacturing and precise alignment. More recently, the capabilities of optical systems have been rapidly expanding with the introduction of adaptive optics technologies. By allowing for the real-time correction of dynamic and spatially complex optical aberrations, adaptive optics enables the pursuit of scientific investigations, military applications, and medical diagnostics that are well beyond the theoretical capabilities of a purely static optical design. However, the dominant adaptive optics technology of piezo electric driven mirrors has tended to be prohibitively expensive. Consequently, adaptive optics were originally considered a specialized and exotic wavefront control device accessible only to a few select research organizations (Laing et al. 1997; Wizinowich et al. 2000) or for use in well-funded military applications (Ealey 1992).

During the previous decade, a major effort has proceeded in the development of MEMS deformable mirror technology as a viable alternative to the piezo electric driven mirrors. While earlier publications (Krishnamoorthy and Bifano 1995; Vodvin et al. 1995) and research concentrated on the development of the MEMS deformable mirror technology itself, we are now witnessing a transition in focus towards the applications of this new and enabling technology (Booth 2007; Potsaid et al. 2005; Tao and Cpo 2007). MEMS deformable mirror products are just now reaching a level of maturity to be considered for inclusion in certain high performance commercial, scientific, and consumer products. And with the potential for low cost, large numbers of actuators, and small aperture sizes, MEMS deformable mirrors are well poised to extend far beyond the current range of applications. However, the barriers for entry are still quite high. Detailed information published on deformable mirror shape correcting capability is often associated with a one-of-a-kind deformable mirror for a special application. And in the case of commercially available deformable mirrors, the shape correcting capabilities of a particular mirror are often provided with respect to maximum stroke for pure Zernike polynomial wavefront correction (Zhou and Bifano 2006), which is often not sufficient for detailed design purposes.
This research seeks to 1) increase understanding of the true capabilities of this new technology and 2) offer effective methods for systematically designing and evaluating a MEMS deformable mirror in a potential optical application.

In a typical MEMS deformable mirror, a membrane (Dayton et al. 2002) is suspended over a pattern of electrostatic actuators, with the possibility of additional microstructures (Bifano et al. 1997) to affect the membrane movement. Because the electrostatic actuators in the MEMS deformable mirrors can only pull on the membrane surface, and because the shape of the membrane itself is determined by the associated membrane mechanics, there are tradeoffs in the achievable amplitude of wavefront correction with respect to spatial frequency, temporal frequency response, and amplitude. Consideration of the specific limitations for each of the different MEMS deformable mirror designs can help guide mirror selection at the early stages of the design process for a given application. And incorporating the shape forming limitations of a given mirror into the design and optimization process as design constraints can ultimately yield higher performance and more optimal system designs. Facilitating proper selection and optimal utilization of MEMS deformable mirrors were the primary motivating factors for this research, which presents and compares two methods for including realistic deformable mirror constraints in the optical design process. The first method combines an optical design software, Zemax, with a finite element modeling software, ABAQUS, to generate realistic deformable mirror profiles. In this first method, a top level supervisory software coordinates data exchange and orchestrates the overall optimization between the two simulation codes. In a second method, a fast approximation model is developed and trained using results from the finite element software. Once trained, this fast approximation model can be included directly in the ZEMAX optical design software such that the deformable mirror actuation voltages can be optimized along with all other parameters in the full optical system. Results are shown for one specific deformable mirror, however, it is anticipated that the underlying approach can be applied to other deformable mirror configurations.

This article first describes a growing need for integrated design in Section 2. A finite element method (FEM) model of a commercial three layer MEMS deformable mirror is presented in Section 3. Section 4 describes a methodology for how the FEM model can be included in an integrated design approach such that information about the deformable mirror shape can be communicated to a commercially available optical design and simulation software, ZEMAX. An alternative approach using a fast approximating model trained on the FEM results is described in Section 5. Section 6 discusses the results, compares the two integrated design approaches, and presents the conclusions.

2. INTEGRATED OPTICAL DESIGN

Most adaptive optics systems (e.g., astronomical telescopes and ophthalmology instruments) are designed to achieve near ideal performance under nominal operating conditions. The adaptive optics element is primarily used to compensate for a time varying disturbance to the wavefront that is external to the optical system itself. While the static optical elements (lenses and mirrors) in such systems must be designed to be compatible with the adaptive optics element with respect to pupil size and conjugate locations along the optical axis, the deformable mirror targets aberrations originating outside the optical system (e.g., atmospheric aberrations for astronomical telescopes and aberrations in the human eye for ophthalmology). For such systems with comparatively little interaction between the optical system and deformable mirror shape forming capabilities, it is convenient to partition the design process into two separate but related tasks. The optical system can be designed without intimate knowledge of the specific shape correcting capabilities of the deformable mirror. And the deformable mirror must be matched to the optical disturbance, e.g., a Kolmogorov model of the atmospheric or statistical model of the human eye.

In contrast to this approach, a new type of optical system is emerging that uses the adaptive optics as an integral component that explicitly interacts with the static optical components in the system for improved performance. Examples include wide field scanning (Potsaid et al. 2003) and variable view angle microscopes (Tao and Cho 2007), foveated (Martinez et al. 2001) and agile field (Graeneiser et al. 2005) scanning telescopes, and projection systems aimed at a consumer market (Svevik et al. 2007). In such systems, the interaction between the deformable mirror and the static optical elements must be considered. In fact, the parameters of the static optical elements (lens or mirror surface profiles, lens thicknesses, glass type, etc.) should be explicitly optimized to the specific shape correcting capabilities of the deformable mirror being used in the system. In this way, the design of the static optical elements effects the deformable mirror shapes required. And conversely, the achievable shapes from the deformable mirror effect the design of the static optical elements, as further optimization of the static optical elements can cancel out aberrations beyond the deformable mirror's shape forming capability. The design methods to accomplish these goals as presented in this paper are based on two high fidelity deformable mirror models.

3. DEFORMABLE MIRROR FINITE ELEMENT MODEL

We consider the 140 actuator MEMS deformable mirror technology from Boston Micromachines Corp. (Watertown, MA) with a nominal stroke of 3.5 |m. Figure 1 shows the principle of deformable mirror actuation and deformation. Figure 2 shows the mirror's actuator layout and corresponding mirror configuration.
and 400 µm actuator spacing. As shown in Figure 1a, the actuators are arranged in a 12 × 12 actuator grid pattern, minus the four corner actuators. A unique characteristic of the Boston Micromachines deformable mirror technology is the inclusion of a third layer in the design as shown in Figure 1b. This third layer is a thin silicon beam that is doubly cantilevered at both ends, attached to the deformable mirror's continuous facesheet by a thin post, and suspended over an electrostatic actuator pad located underneath the actuator beam's bottom surface (Bifano et al. 1997). By charging the pad below, an attractive electrostatic force pulls down on the actuator beam and attached facesheet surface. If the voltage is removed from the actuator pad, the stiffness of the actuator beam acts as a restoring force, pushing upwards on the facesheet surface. A typical usage scenario is to bias all of the actuators to a voltage that deflects the mirror to approximately half of the total stroke capability. In this way, reducing the voltage to an actuator results in a localized positive surface deflection and increasing the voltage to an actuator results in a localized negative surface deflection. A wide range of surface shapes can be generated in this manner and the third layer design is advantageous in its ability to generate fairly large amplitude high spatial frequency corrections. However, because of the associated membrane and actuator mechanics, there are non-ignorable actuator coupling effects between neighboring actuators, as well as a nonlinear response due to the large deflections involved and electrostatic actuation principle.

Several approaches have been proposed for modeling this type of mirror. In early developmental work of this mirror technology, Bifano et al. proposed a finite difference approach (Bifano et al. 1997). Both Vogel (Vogel and Yang 2006) and Mrozinski (Mrozinski et al. 2007) develop a model for the purpose of open-loop control of the mirror for astronomical telescope applications. All these approaches assume a simple linear plate model or small deflections. Our requirements are different than previous model development in that a highly accurate model for the purpose of optical system design is required. Rather than operating well within the stroke capabilities of the mirror, where a static FEM stiffness matrix is sufficient, evaluation of a mirror in a new design requires stressing the mirror to performance limits. It is therefore advantageous to model the nonlinear effects of geometry change, as well as the nonlinear effects of the actuation principle. Furthermore, being able to represent stable geometries in the surface profile (e.g., actuator imprinting, nonlinear coupling, etc.) provides a more realistic performance scenario. For these reasons, we have chosen to model the mirror in a commercially available finite element modeling code (ABAQUS). Specific deformable mirror geometry information was obtained from Boston Micromachines for this purpose.

The MEMS deformable mirror from Boston Micromachines has complex geometry with many high aspect ratio features. Attempting to model all of the geometry and electrostatic actuation in a single simulation results in a large model with long simulation times. To circumvent this limitation, we separate the model into an actuator model and membrane model. After characterizing the actuator itself, a mathematical approximation to the actuator performance is applied to a membrane model to reduce the time required for the mirror analysis. A more detailed description of the finite element modeling of this mirror can be found in Potsaid and Wen (2007). Figure 2 compares the results of the FEM simulation to experimental mirror profile data obtained with a ZYGO NewView 6000 series white light interferometer.

A 3 × 3 array of actuators was set to 250 V and the resulting mirror profile is shown. The maximum deflection of the actual mirror (3.7–3.8 µm) and the FEM simulation (3.9 µm) are close and the deformed shapes are quite similar. Note that in the experimental data shown, the unactuated mirror shape has been subtracted from the deformed shape to remove surface irregularities resulting from residual manufacturing induced stresses in the deformable mirror structures. These residual stresses were not modeled in the FEM simulation, but would have to be taken into account in practice. We deemed this simplification acceptable, considering the difficulty of modeling this artifact when each mirror manufactured has a slightly different residual stress pattern and that the emphasis of the research was in developing an integrated design methodology.

4. FINITE ELEMENT MODEL DESIGN APPROACH

The first integrated approach seeks to design and optimize both the static optical elements and the deformable mirror concurrently. This is accomplished by connecting an optical simulation to a finite element model of the deformable mirror and optimizing both together (Potsaid et al. 2006). A top level coordination implemented in MATLAB exchanges information between the two codes as shown in Figure 3.

There is a two-way communication between the optical software and deformable mirror simulation software. The optical simulation contains a local representation of the deformable mirror shape and communication to the DM code a new deformable mirror shape that is preferable. The DM code then optimizes the actuator voltages, abiding by actuator saturation, to generate a feasible deformable...
mirror shape that most closely matches the requirements of the optical simulation needs. The resulting mirror shape can then be communicated to the optical code as a feasible target and the process iterated. Thus, the concurrent optimization is accomplished by:

- Establishing a communications path between MATLAB, ZEMAX, and ABAQUS as shown in Figure 3 and representing the deformable mirror shape with a set of basis functions and associated basis coefficients.
- Introducing a relaxation between the basis coefficients (and hence the deformable mirror surface profile) within each subsystem.
- Coordinating all simulation and optimization codes to achieve the overall design objective.

### 4.1. Deformable Mirror Basis Parameterization

The finite element model takes as input the voltages applied to the deformable mirror and outputs a surface profile. Thus, a high-fidelity approximation to the membrane surface profile, $S$, can be determined for a given set of actuator voltages, $V$:

$$ S = f(V) $$

An approximation to the deformable mirror shape, $\tilde{S}(x,y)$, can be represented as the superposition of a set of basis, $b_i$, with each of the $n$ basis having an associated basis coefficient, $\phi_i$. The surface sag of the deformable mirror is then calculated as the linear superposition of the $n$ basis shapes added to the nominal deformable mirror shape as shown in Figure 4 and calculated as:

$$ \tilde{S}(x,y) = b_{nom}(x,y) + \phi_1 b_1(x,y) + \phi_2 b_2(x,y) + \cdots + \phi_n b_n(x,y) $$

(2)

where $b_{nom}(x,y)$ is the shape of the deformable mirror with all actuators biased to a nominal set of voltages and $\Phi = [\phi_1, \phi_2, \ldots, \phi_n]^T$.

Using this parameterization of the deformable mirror shape, information about the deformable mirror can be communicated by exchanging only the $n$ scalar basis coefficient values, $\Phi$, between subsystems (each subsystem has a local representation of the basis shapes, which do not change themselves during the optimization).

### 4.2. Optimization Relaxation

The ability to guide the shape of the deformable mirror as required by the optical subsystem to improve overall system level performance is enabled by introducing a relaxation between the value of the basis coefficients within ZEMAX and the value of the basis coefficients within the deformable mirror simulation. However, although the values of the coefficients can differ between ZEMAX and the DM simulation, a consistency function is included in the ZEMAX merit function to ensure eventual agreement between the FEM surface profile and the profile represented with ZEMAX at the end of the overall system optimization. It is through this consistency function that the realistic deformable mirror constraints are incorporated, including actuator voltage saturation, actuator coupling between neighboring actuators, and nonlinearities in the membrane mechanics and electrostatic actuation. The consistency function, $Z$, can be written as:

$$ Z = ||\Phi_{DM} - \Phi_{ZEMAX}||^2 $$

(3)

where $\Phi_{DM}$ is the value of the basis coefficients fit to the FEM result and $\Phi_{ZEMAX}$ is the value of the basis coefficients used within ZEMAX to construct the deformable
mirror shape used for simulation, \( \beta \) is an overall weighting factor that governs how close the agreement between the ZEMAX simulation and FEM surface profiles must be. Using a small \( \beta \) for the first iterations allows for a significant difference between the \( \Phi_{\text{FEM}} \) and ZEMAX values, which can increase the speed at which ZEMAX affects the deformable mirror shape, but can allow an inflexible mirror profile shape to be simulated. Higher values of \( \beta \) ensure that the mirror profile simulated within ZEMAX is feasible, but slows the rate at which the deformable mirror shape can change from iteration to iteration.

### 4.3. Optical System Optimization

The goal of the optical system optimization is to minimize the wavefront error while abiding by all other optical design constraints and design intent as represented in the ZEMAX merit function. The deformable mirror (DM) is represented in ZEMAX by creating a user-defined surface. ZEMAX allows a user to write custom code to perform ray tracing for non-standard surfaces and to link this code to ZEMAX with a dynamic-link library (DLL). A bicubic interpolation was used to represent the DM surface because it is continuous in the first derivative across grid points and cell boundaries, which is desirable because surface normals need to be calculated for proper ray tracing. The user-defined surface was configured such that the basis coefficients were accessible as variables, which could be included in the merit function with \( \Phi_{\text{DM}} \) as the target value, and \( \beta \) as the operand weight. The variables defining the deformable mirror shape can then be optimized with all other variables within ZEMAX. The Orthogonal Descent optimization algorithm was used within ZEMAX to help manage some of the noise issues associated with the FEM simulation and bicubic interpolation scheme.

### 4.4. Deformable Mirror Optimization

The goal of the deformable mirror optimization is to adjust the voltages in the finite element simulation to generate a deformable mirror shape that most closely matches the shape communicated by ZEMAX (in terms of basis coefficient values). Afterwards, the deformable mirror optimization determines a set of basis coefficients that best represents the resulting FEM simulated mirror profile to be returned to ZEMAX as a set of target coefficients representing a feasible mirror profile. In both cases, the optimization approach was influenced by Marquardt approximations, which shows that the Strehl ratio can be maximized by minimizing the RMS wavefront error, \( \sigma \), and is defined as Strehl \( \equiv e^{-2\sigma^2/\lambda^2} \), with \( \lambda \) the wavelength of light. The deformable mirror optimization thus attempts to minimize the RMS surface error during both the FEM optimization and basis fit steps.

Consider \( S_{\text{FEM}} \) to be the desired deformable mirror surface profile as communicated by ZEMAX. All of the basis, \( h(x,y) \) are 2-D in representing the surface profile, but the same information can be written as an equivalent 1-dimensional vector by stacking the columns of \( h(x,y) \) as \( h \), where the caret symbol designates a vector representation of the matrix. A matrix, \( F \), can then be constructed with each column representing a basis shape. With \( h \) a basis, \( F \) is defined as:

\[
P = [h_0 \ h_1 \ h_2 \cdots h_n]
\]

If we define \( g = \delta \), \( Q \), as the vector of these linear perturbed voltages, and the surface error, \( E \), to be \( E = S_{\text{FEM}} - S_{\text{FEM}} \), the minimization simplifies to:

\[
\frac{1}{2} \| E - Fg \|^2
\]

The least squares solution to this set of overconstrained linear equations (the number of grid points is greater than the number of actuators) is:

\[
Q^* = (F^TF)^{-1}F^TE
\]

where \( F^TF \) is the Moore-Penrose matrix inverse of \( F \). However, the actual system is nonlinear, so the process is repeated with the actuator voltages updated as:

\[
V_{n+1} = V_n + \alpha n^e
\]

where \( \alpha \) is the step size along the search direction defined by \( n^e \). A step size of \( \alpha = 0.85 \) was used in this research. Optimizing the FEM mirror to fit the desired shape requires an iterative approach to manage the nonlinearity. However, once the FEM mirror surface is generated, the target basis coefficients to be communicated to ZEMAX representing a feasible shape can be found by a direct least squares fit of the basis to the FEM mirror profile.

### 4.5. System Level Coordination

The system level coordination was implemented in MATLAB and data was exchanged between MATLAB, ZEMAX, and ABAQUS through shared files and Windows Dynamic Data Exchange (DDE). With the basis shapes already generated and stored beforehand, the optimization process is described in Algorithm 1.

**Algorithm 1. Optimization Algorithm**

1. set all basis coefficients to zero in ZEMAX
2. set all target coefficient values to zero in ZEMAX
3. while not converged and maximum iterations not exceeded do
4. perform Optical System Optimization in ZEMAX
5. transfer resulting basis coefficient values to MATLAB
6. perform Deformable Mirror Optimization with MATLAB executing ABAQUS simulations
7. perform fit of basis to resulting FEM profile in MATLAB
8. transfer best fit basis coefficients to ZEMAX from MATLAB as target values
9. end while
4.6. Finite Element Design Results

For the simulated results, we consider the simple case of defocus correction with the deformable mirror. While the Boston Micromachines deformable mirror with 140 actuators is capable of producing much higher order aberrations correction (Zhou and Bilano 2006), we expect to be better able to interpret the effects of actuator saturation, nonlinearity, and actuator coupling on the convergence properties of the optimization by concentrating on this low order aberration first. Observations and intuition gained from these experiments will guide future development of the design methodologies, which will be demonstrated with case studies covering a range of practical optical system designs.

The optical system layout is shown in Figure 5. A simple focusing lens was designed to exhibit near ideal optical performance at a nominal focal position (ZEAMAX reports less than 2.0 waves peak-to-valley wavefront error at the 550 nm wavelength used in this study). The focusing lens accepts a collimated beam of light which has previously reflected off the deformable mirror (note that this system does not include the pupil relay optics that are an integral component of most adaptive optics imaging systems). The beam was chosen to be 3.6 mm in diameter as to perfectly span a 10 x 10 actuator grid at 400 µm actuator spacing as illustrated in Figure 5. For the 12 x 12 actuator grid of the Boston Micromachines deformable mirror, this is the largest diameter beam resulting in the outermost actuators at the beam perimeter being surrounded by a full set of eight neighboring active actuators. Intuitively this seems like a logical guideline for general use of this type of mirror and we have found through varying the pupil diameter that this size seems to offer a nice compromise between maximizing the number of actuators used and limiting the detrimental edge effects. However, we expect the optimal active mirror surface to be application specific and will investigate the effect of non-circular projected apertures caused by the angle of incidence on the mirror surface.

Assuming the deformable mirror to be perfectly flat, defocusing the system by 0.025 mm results in 7.1 waves peak-to-valley wavefront error as compared to the nominal focused value of 2.0 waves at a 0.5 numerical aperture. Inserting the model of the deformable mirror into the optical system with all actuators at 180 V results in 7.2 waves peak-to-valley. The associated Strehl ratio at these operating conditions is 0.007, which is plotted as the first iteration data point in Figure 6. Successively applying the optimization algorithm described in Section 4 generates improving performance as demonstrated by increasing Strehl values. Notice in Figure 7, which shows the associated voltages applied to the mirror in the FEM simulation, that several actuators saturate at the 0 V and 250 V drive limits. By imposing these voltage limits in the FEM simulation, the effect of voltage saturation is captured in the optimization process and conveyed to the optical configuration as limits on the target values of the basis coefficient values. After eight iterations, the Strehl has exceeded the 0.8 value, considered the diffraction limit (shown on the plot as a dashed line). However, the Strehl values for iterations 1-8 were calculated using the basis representation of the deformable mirror shape contained in the user defined ZEAMAX surface. This shape is referenced to the nominal mirror configuration, i.e., $b_{opt}$ in Eq. (2) is the mirror shape with 180 V on all actuators. With the large deviation in actuator voltages from this nominal condition, an error develops between...
the FEM generated mirror profile and the mirror profile represented with the basis reconstruction. Indeed, the Strehl drops down to 0.76 (as indicated by the "x" in Figure 6), for the eighth iteration configuration if the user defined DM surface is replaced by a ZEMAX grid sag surface and the direct results of the finite element code imported into ZEMAX. For this reason, the nominal shape, \( A_{\text{nom}} \) in Eq. (2), is updated with the actual FEM result of iteration eight and all of the basis coefficients reset to zero. The optimization algorithm is then continued using the updated user defined DM surface. After 15 total iterations, a Strehl value of 0.81 is achieved (as indicated by an "o" in Figure 6), which is simulated by importing the actual FEM code result into the ZEMAX grid sag surface to eliminate all basis approximations.

Figure 8a shows the resulting deformable mirror shape after the optimization process and Figure 8b shows the associated wavefront error. During the optimization, it was difficult to correct the wavefront much beyond the diffraction limit in this specific testing scenario. The largest aberration is seen along the outer edge of the wavefront, while the central region is well corrected. Considering that each FEM simulation takes approximately 20 minutes to complete, the 18 iterations shown in Figure 6 take nearly 7 hours. Although the deformable mirror shape is very accurate, the long cycle time is clearly a shortcoming of the optimization approach.

In practice, the rate of convergence slows considerably when actuator saturation occurs, and it is likely that the optimization will result in a sub-optimal solution for reasonable design time frames.

5. DEFORMABLE MIRROR FAST APPROXIMATION MODEL

As described in Section 3, the finite element model is capable of generating highly detailed mirror surface predictions for a given set of input voltages. For optical system design purposes, however, the finite element approach suffers from the following shortcomings and disadvantages.

- The long finite element simulation times prevent the high number of iterations required for effective optical system design and optimization.

To address the limitations of the finite element and multidisciplinary design optimization approach, we developed a fast approximation model for the deformable mirror. This model combines the attractive attributes of a very short evaluation time with an accurate and high fidelity mirror surface approximation, including actuator coupling and saturation effects. Furthermore, encapsulating the model in a fully self-contained ZEMAX user defined surface eliminates the need for auxiliary software (i.e., MATLAB and ABAQUS), as well as the complexity and overhead of the integrated FEM approach described in Section 4.

5.1. Fast Approximation Model Structure, Computation, and Training

The fast approximation model represents the shape of the deformable mirror as the superposition of a set of basis functions added to the nominal deformable mirror shape when biased at half deflection, the same as in Eq. (2). However, coefficients for the basis functions, \( \Phi \), are calculated as nonlinear functions, \( g \), of the actuator input voltages, \( V \).

\[
\Phi = g(V)
\]  
(7)

It is important that \( g(V) \) and the basis functions be selected to represent the actuator coupling, nonlinearity, and saturation effects of the deformable mirror. In the Boston Micromachines Deformable mirror, the actuator coupling effect is quite local, as can be seen in Figure 2. Indeed, the deflection of the mirror at a particular actuator location is primarily determined by the voltage applied to the actuator and the voltage applied to the actuator's immediate neighbors. This suggests that a zonal approach using the actuator influence functions (rather than a Zernike basis for example) would be suitable. Each of the 140 basis functions (see Figure 4) was filtered with a 7 x 7 gaussian filter to remove a high frequency FEM simulation noise that became apparent when the basis where scaled with large coefficients. Filtering in this manner preserved the dominant shape of the basis function, with the maximum difference between the filtered and unfiltered shape being two orders of magnitude smaller than the maximum amplitude of the basis itself. Synthesis of an appropriate nonlinear mapping function then proceeded as follows:

1. Fit the basis functions themselves to each deformable mirror profile. This generates the optimal set of basis coefficients for a given deformable mirror profile, while ignoring all actuator effects.
2. Generate and fit the nonlinear approximating function to best reproduce the optimal set of basis coefficients (as determined in step 1 above) from the corresponding actuator voltage input data.

The training data consisted of a set of 59 random deformable mirror shapes (where each voltage was randomly chosen to be in the range of 0-250) in addition to the maximum and minimum deflection cases of zero volts on all actuators and 250-volts on all actuators. Consider $S_{FEM}$ to be the deformable mirror surface profile generated from the ith finite element simulation. All of the basis, $b_i(x,y)$ are 2-D in representing the surface profile, but the same information can be written as an equivalent 1-D vector by stacking the columns of $b_i(x,y)$ as $\tilde{b}_i$, where the caret symbol designates a vector representation of the matrix information. A matrix, $P_i$, can then be constructed with each column representing a basis shape. With a basis, $P$ is defined as $P = [b_1 b_2 \ldots b_n]$.

The solution to minimize the squared error for the ith data set is given by

$$\Phi_i^* = P_i S_{FEM}^* - \tilde{b}_{i \text{data}}$$

where $b_{i \text{data}}(x,y)$ is the nominal shape of the deformable mirror, the same as in Eq. (2), $S_{FEM}^*$ is the vector representation of the column stacked surface profile, $P_i$ is the optimal set of basis coefficients for the ith data set. Processing all data sets ($i = 1 \text{ to } n$) results in an optimal set of basis coefficients corresponding to each set of input voltages, from which an appropriate nonlinear mapping can be found.

A polynomial basis representation is often used as an approximating function because of its simplicity and rapid computation times. When cross-coupling terms are included, such a basis can be an effective approximating function for many practical design problems (Myers 1971). Through experiment, it was determined that a fourth order polynomial expansion including a subset of the cross-coupling terms worked well as an approximating function to the full finite element model. Including terms based on the average voltage of the nine neighboring actuators helped to address a residual DC bias in the actuator deflection. Thus, each basis coefficient was determined by the voltage of the associated actuator and the eight neighboring actuators. Referring to Figure 9, for any ith actuator, $U_i$ is defined as the voltage of the ith actuator and the eight neighboring actuators are designated as $U_1-U_8$ and $U_9-U_{16}$, which values are of course update as a different actuator location is considered in the full array. Thus, for the ith actuator, the three primary components (fourth order polynomial coupling terms, actuator cross-coupling terms, and average actuator voltage terms) of the approximating function are as follows.

$$A_i^{4\text{th}} = \sum_{j=1}^{8} U_j c_{i,j} + (U_j/100)^2 c_{i,j2} + (U_j/50)^4 c_{i,j4}$$

$$A^{10\text{th}} = \sum_{j=1}^{8} U_j c_{i,j} + (U_j/100)^2 c_{i,j2} + (U_j/50)^4 c_{i,j4}$$

where $U$ represents the mean average value of $U_1$ to $U_8$ and $\|U\|_p$ is the p-norm of the $U$ vector. Notice in the above that the voltages are scaled by a constant to reduce numerical computation problems as the third and fourth order terms can grow quite large.

The basis coefficient is then calculated as $\Phi_i = A^{10\text{th}}(V) + A^{4\text{th}}(V) + A_{\text{data}}(V)$. Separating out the coefficients, $c_{ij}$ and writing in matrix form, the basis coefficient can be calculated as

$$\Phi_i = d_i U_i C$$
where \( d_i(U) \) contains the polynomial and average voltage basis terms for the \( i \)th actuator and \( C = [c_1 \ldots c_N] \). An assumption is made that all actuators behave similarly as a function of the neighboring voltages, regardless of location on the deformable mirror surface. In order to fit one set of basis coefficients to all \( n \) actuators of \( m \) sets of training data, the data is arranged by stacking the optimal basis coefficients and basis functions as \( \tilde{D} = [\tilde{d}_1; \tilde{d}_2; \ldots; \tilde{d}_m] \), where each \( \tilde{d}_i \) is obtained from Eq. (8). Similarly, the basis coefficient functions can be stacked to form \( \tilde{D} = [D_1; D_2; \ldots; D_m] \) for all \( m \) sets of training data and \( D_i = [d_1; d_2; \ldots; d_n] \) for all \( n \) actuators. The generalized set of polynomial coefficients that minimize the squared error of the basis coefficients is given by,

\[
C^* = \hat{D}^\dagger \bar{D}
\]

where \( \dagger \) signifies the Moore-Penrose matrix inverse.

Given a set of actuator voltages, the reconstructed basis coefficients, \( \hat{C} \), can then be rapidly obtained by \( \hat{C} = \hat{D} \hat{D}^\dagger C \). To verify the fast evaluation model, a set of actuator voltages not used in the training process were run through the finite element simulation and fast evaluation model for comparison. As shown in Figures 10a and 10b, it can be seen that the fast approximation model and the finite element solution agree very closely, with the resulting error shown in Figure 10c.

5.2. Fast Approximation Model Usage

The fast evaluation model described in Section 5 was translated into the C programming language and incorporated into a ZEMAX user defined surface dynamically linked library (DLL). In this way, the voltages to the deformable mirror model can be represented as variables in a ZEMAX model and accessed through the “Extra Data Editor.” Different than in Section 4, the value of the variable represents the voltage applied to the actuator, allowing voltages constraints to be applied directly to the variables by means of the merit function definition within ZEMAX. Lower bound voltage limits were included using the XDGT (Extra Data Value Greater Than) operand with a target value of 0.0 and weight of 1.0 for each of the 140 actuator voltages. Upper bound voltage limits were included using the XDLT (Extra Data Value Less Than) operand with a target value of 250.0 and weight of 1.0 for each of the 140 actuator voltages. As is typical in optical design, the optimization problem contains local minima and there is always a tradeoff between the computational burden and the fidelity of the merit function containing the design objective. For efficient optimization with the fast approximation model in ZEMAX, it was found that starting the optimization using a gaussian quadrature (GQ) pupil integration method resulted in an initial rapid convergence, but even with the maximum of 20 rings and two arms for the ray aiming, the GQ method undersamples the wavefront because of the high number of deformable mirror actuators. Switching to a 32 x 32 grid sampling is slower, but provides adequate sampling of the wavefront. It was also found that the orthogonal descent (OD) algorithm in ZEMAX worked better than the damped least squares (DLS) algorithm, possibly because of the noise in the basis shapes or the high number of variables in the optimization. During the optimization, it is inevitable that local minima are encountered, catching the optimization at a false minimum. Periodically switching to minimize the RMS spot size versus the RMS wavefront error can perturb the system enough to escape local minima. Then returning to minimizing the RMS wavefront often produces a superior result. Alternatively, the Hammer Optimization (random walk based optimizer) in ZEMAX can often find a better solution, but sometimes the optimization must be left to run overnight.

5.3. Fast Approximation Model Results

Consider the defocus scenario outlined in Figure 5, but operating at a 0.06 numerical aperture. The static optical elements in the objective lens are designed to be defocus limited at the nominal focus position. Using a flat mirror in place of the deformable mirror in ZEMAX, the amount of defocus was varied from 0 mm to 3.5 mm, with the results for peak to valley wavefront error shown in Figure 11. In this plot, it can be seen that the peak to valley wavefront error exceeds the expected range of the DM stroke \( \Delta_{net}/h_a \), where \( \Delta_{net} = 3.6 \mu m \) and \( h_a = 550 \) mm) at a little over 3 mm of defocus. After inserting the fast approximation model user defined surface and applying the optimization technique described in Section 5, the peak to valley wavefront was reduced significantly with the deformable mirror, as compared to the uncorrected results shown in Figure 11. Noting that relatively small regions of the wavefront near the edges contribute disproporionally to the peak to valley wavefront error, a measure more representative of the system performance is the Strehl ratio, which is shown in Figure 12. A Strehl ratio above 0.8 is considered to be diffraction limited (the goal of most optical system design) and as expected, the Strehl ratio drops off significantly to below this.
threshold when the defocus exceeds 2.7 mm, but remains uniformly quite high within this defocus range. Further insight can be gained from Figure 13, which shows the resulting deformable mirror profiles and associated wavefronts for the 2.5 mm defocus case and 3.1 mm defocus cases. In the 2.5 mm defocus case shown in Figure 13a, there is little to no actuator saturation and the wavefront is well corrected, showing only small residual bumps from the influence functions. However, in the 3.1 mm defocus case shown in Figure 13, there is considerable saturation, which manifests itself as a clearly visible wavefront error in the center and edges of the pupil.

6. CONCLUSIONS

As the cost of deformable mirrors and other adaptive optics technologies decrease, it is envisioned that a wide range of optical systems will benefit from their usage. However, as conceptually simple as this new and emerging technology seems, the true capabilities and performance of the adaptive optics elements are not widely understood as there are limitations to their shape correcting capabilities. The goals of this research are to provide tools and methods (1) for evaluating the deformable mirror technologies in new applications and (2) to provide effective design methods for incorporating adaptive optics in new designs such that the optical system itself can be optimized to the shape correcting capabilities of the specific deformable mirror in the design.

Two methods for integrating a deformable mirror model in the design process are presented within the context of defocus case studies. A first method that uses a high fidelity finite element model in an integrated design process produces accurate results, but because of the long evaluation time of the FEM model, requires extremely long times to execute. To address this shortcoming, a fast approximating model is developed, which can be directly incorporated into existing commercially available optical design software. Executing in a fraction of a second, this approximation model can be effectively used for integrated optical design. In this way, both the optical system parameters and the deformable mirror voltages can be optimized concurrently to yield a further increase in system performance. The basic viability of the approach is demonstrated in this paper and future work will be to verify against
Experimental obtained deformable mirror profile data. Further work will investigate replacing the FEM model with experimentally obtained data to eliminate the FEM model and all associated approximations entirely.

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