Motion and Force Control of Multiple Robotic Manipulators*

JOHN T. WEN† and KENNETH KREUTZ-DELGADO‡

The motion and force control problems are decoupled for multiple manipulators cooperatively handling an object. Stable and robust motion and force control algorithms, developed and analyzed in this framework, are compared to illustrate their advantages and disadvantages.

Key Words—Robots; control applications; stability; Lyapunov methods; force control; robot control.

Abstract—This paper addresses the motion and force control problem of multiple robot arms manipulating a cooperatively held object. A general control paradigm is introduced which decouples the motion and force control problems. For motion control, different control strategies are constructed based on the variables used as the control input in the controller design. There are three natural choices; acceleration of a generalized coordinate, arm tip force vectors, and the joint torques. The first two choices require full model information but produce simple models for the control design problem. The last choice results in a class of relatively model independent control laws by exploiting the Hamiltonian structure of the open loop system. The motion control only determines the joint torque to within a manifold, due to the multiple-arm kinematic constraint. To resolve the nonuniqueness of the joint torques, two methods are introduced. If the arm and object models are available, an optimization can be performed to best allocate the desired, and effector control force to the joint actuators. The other possibility is to control the internal force about some set point. It is shown that effective force regulation can be achieved even if little model information is available.

1. INTRODUCTION

Coordination between multiple manipulators in robot task execution has received increasing attention in recent years. It is the basic technology for applications that are beyond the capability of a single arm, for example, the manipulation of massive and bulky objects, handling of a flexible payload, grasping, dextrous multiple-finger manipulation, and even multi-legged locomotion. The control of multiple robots represents a significant increase in complexity over the single arm case due to the dynamical interaction between multiple mechanisms (arms, payload, robot base, environment, etc.). If this problem is not carefully addressed, highly undesirable consequences may result, such as reduced life span of components, damaged parts, or task failure.

Many low level servo control schemes of multiple-robot systems have been proposed. A master/slave scheme is proposed in Zheng and Luh (1985) where one arm is under position control and the others are subject to compliant force control to maintain the kinematic constraint. In Arimoto et al. (1987), the proportional-derivative (PD) (for a set of generalized coordinates) plus gravity scheme is generalized from the single arm case (Takegaki and Arimoto, 1981) to the multiple-arm case. There are also many other strategies centered around feedback linearization with respect to a generalized coordinate in the task space (Hayati, 1986; Tarn et al., 1987; Khatib, 1988). The purpose of this paper is to present a unified perspective on the motion and force control problem for multiple-arm systems and to propose a framework for the stability analysis.

We assume that the arms, the payload, and the grasps are all rigid. The rigid grasp assumption means the constraints on the arms are holonomic. Other types of grasps, such as point contact, soft finger, etc. are currently under investigation. With the rigidity assumption, the composite contact force from all arms can be decomposed into two orthogonal components: one effects motion of the held object and the other produces an internal force (called the squeeze force). As a consequence, the motion and control problems become decoupled (in one direction) in the following sense: "Force control does not affect object
motion, although object motion does affect the internal force (due to the inertial, d'Alembert force)". This motivates the following control design philosophy: "Design a stable motion control law without the consideration of force control. Then design a stable force control law by treating the inertial force as a perturbation (which is independent of the force control)". For the motion control problem, the choice of the variable considered as the control input in the control design results in different control architectures. We consider the following three obvious choices.

(1) The acceleration of a generalized coordinate. This is called the feedback linearization approach.

(2) The composite tip force of all the arms. This is called the arms-as-actuators approach.

(3) The joint torques of all the arms. This is called the full dynamics approach.

In the first two approaches, the desired control input is realized from the joint torques via a feedforward compensation which requires real time measurements (joint position and velocity) but has no error correction function. The advantage of these approaches is that the control design involves a much simpler model (decoupled double integrators in the first case, and Newton's and Euler's equations in the second case). However, full model information is required for their implementation; the related computational and robustness issues remain to be fully explored. In the full dynamics approach, the complete nonlinear robot dynamical model needs to be considered during the control design stage. By exploiting the fact that the system is a Hamiltonian system and the full state information is assumed available, we proposed an energy Lyapunov function approach (a generalization of the single arm case in Takegaki and Arimoto [1981]) to show the closed loop global stability of a class of relatively model independent control laws for (1) set point control and (2) set point control with transient shaping. These control laws share the common structure of proportional and derivative feedback with gravity compensation. Depending on the parameterization of the artificial potential energy in the Lyapunov function, different feedback variables are possible. Here, we consider three natural cases: joint variables, tip variables and generalized coordinates. In all three cases, the asymptotic velocity tracking error is zero. However, only in the case of generalized coordinate does the position tracking error also tend to zero. In the first two cases, we can only conclude that the position tracking error lies in a manifold on which the tip forces produced by the arm position errors balance with each other. We call this manifold the Jam Manifold. Even if the desired configuration satisfies the multi-arm kinematic constraint, the Jam Manifold may consist of other jammed configurations. For this case, we can only at present show that if the proportional gain of one arm is much greater than the gains of all other arms, then the Jam Manifold only consists of the desired configuration.

Because of the multiple arm kinematic constraint, there is an actuation redundancy in any motion control strategies for fully actuated arms. To resolve the underdeterminacy of the joint torques, two methods are possible. An optimization problem can be posed to distribute the required motion control force at the arm end effectors to the joint actuators subject to the internal force and actuator saturation constraints. Another method is to control the internal force about a set point. Both feedforward or feedback (if force/torque information is available for all arms) strategies can be used. The feedback scheme has better robustness properties and can achieve tight force control if a high integral feedback gain is used.

Though this paper revolves around multiple-arm control, the hybrid force/position problem for a single arm in contact with a rigid environment is of exactly the same form. Therefore, all of the results here hold for that important case also.

This paper is organized as follows. The multiple-arm model and the move/squeeze decomposition of the composite contact force is stated in Section 2. The motion control problem is treated for all three approaches in Section 3 with the most emphasis placed on the full dynamics approach. The force control problem is discussed in Section 4. Section 5 presents some results concerning the Jam Manifold. Simulation results for two three-link planar arms are shown in Section 6.

2. MODEL FOR MULTIPLE-ARM SYSTEMS

Throughout this paper, we assume that the arms and the held object are rigid and the grasp between the arms and the object is also rigid (i.e. no relative motion at the contact). Other models of multiple-arm systems sometimes insert a spring in the last link of each arm to simulate the effect of force/torque sensors. We feel that because the internal spring in the force/torque sensor is sufficiently rigid (implying small displacement) and the anticipated force transient is sufficiently benign (due to our force
controller), our infinite rigidity assumption is a reasonable approximation. The flexibility effect does affect the feedback gain selection, and more will be discussed on this topic in Section 4. A consequence of the rigidity assumption is that a (squeeze) force can be applied without causing any motion.

With the stated assumptions, the equation of motion for (Rodriguez and Kreutz, 1988):

\[ M(q)q = τ − C(q, ˙q)q − k(q) − JT(q)f \]  
\[ M_α = A^Tf + b_εk_c \]  
\[ a = A_α + a = ˙J(q)q + J(q)q = ˙v \]  
\[ v = Av_ε = J(q)q \]  

(1) (2) (3) (4)

The arms are numbered from 1 to m, and arm i is assumed to have Ni links. The vector q denotes the stacked vector \( [q_1^T, \ldots, q_m^T]^T \), where \( q_i \) is the joint angle vector for arm \( i \). The same stacked representation is also used for \( ˙q, ˙q, τ, k, f, a, v \) and \( b \). The matrix \( M \) is a block diagonal matrix with ith diagonal block \( M_i \), the inertia matrix of arm \( i \). This block diagonal representation is also used for \( C, J \) and \( J \). If the force and torque at a frame are \( F \) and \( T \), respectively, we call the stacked variable \( [T^T, F^T]^T \) the spatial force. Similarly, if the linear velocity and angular velocity of a frame in a rigid body are \( V \) and \( W \), respectively, we call the stacked variable \( [W^T, V^T]^T \) the spatial velocity. Spatial acceleration is defined similarly. The definitions of the variables are summarized in Table 1 (object frame \( C \) denotes a coordinate frame fixed with respect to the held body).

<table>
<thead>
<tr>
<th>Table 1. Definition of Variables</th>
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<tbody>
<tr>
<td>( q ) = composite joint angle vector</td>
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<tr>
<td>( ˙q ) = composite joint angular velocity vector</td>
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<tr>
<td>( ˙q ) = composite joint angular acceleration vector</td>
</tr>
<tr>
<td>( τ ) = composite joint torque vector</td>
</tr>
<tr>
<td>( k ) = composite arm gravity load vector</td>
</tr>
<tr>
<td>( f ) = composite arm tip spatial force vector</td>
</tr>
<tr>
<td>( x ) = composite arm tip position/orientation (assuming a three-parameter representation)</td>
</tr>
<tr>
<td>( v ) = composite arm tip spatial velocity vector</td>
</tr>
<tr>
<td>( a ) = composite arm tip spatial acceleration vector</td>
</tr>
<tr>
<td>( M ) = composite inertia matrix</td>
</tr>
<tr>
<td>( C ) = composite centrifugal and Coriolis force matrix</td>
</tr>
<tr>
<td>( J ) = composite Jacobian (from joint velocity to arm tip spatial velocity)</td>
</tr>
<tr>
<td>( j ) = composite Jacobian derivative</td>
</tr>
<tr>
<td>( x_\epsilon ) = spatial position/orientation of object frame ( C ) (assuming a three-parameter representation)</td>
</tr>
<tr>
<td>( v_\epsilon ) = spatial velocity of object frame ( C )</td>
</tr>
<tr>
<td>( a_\epsilon ) = spatial acceleration of object frame ( C )</td>
</tr>
<tr>
<td>( b_\epsilon ) = spatial force at the object frame ( C ) due to the centrifugal and Coriolis acceleration</td>
</tr>
<tr>
<td>( k_\epsilon ) = spatial force due to gravity at the object frame ( C )</td>
</tr>
<tr>
<td>( A ) = Jacobian transformation from object frame ( C ) to arm tip frames</td>
</tr>
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</table>

Equation (1) is the standard robot dynamics equation where \( f \) represents the force exerted by the arm tips on the held object. Equation (2) governs the dynamics at the object frame \( C \). It is the composite of Newton's and Euler's equations of motion. Equation (3) relates the arm tip spatial accelerations to the spatial acceleration at object frame \( C \) and also to the joint velocity and accelerations. Equation (4) relates the arm tip spatial velocity to the spatial velocity at object frame \( C \) and also to the joint velocity.

These equations can be combined to solve for the contact force, \( f \), as

\[ f = (AM_c^{-1}A^T + JM_c^{-1}J^T)^{-1} \]
\[ \times [\dot{q} − AAM_c^{-1}(b_c + k_c) \\
+ JM_c^{-1}(τ − Cq − k)] \]  

(5)

\( f \) is uniquely solvable if and only if \( [J \ A] \) has full rank. It was shown in Milman et al. (1988) that if the manipulator system is kinematically parameterized by the Denavit–Hartenberg parameters and base positions of the arms, then \( [J \ A] \) having full rank at every kinematically feasible configuration is a generic property. Hence, we will assume that \( f \) is uniquely solvable.

In (2), \( A^T \) is of the form

\[ A^T = [A_1^T, \ldots, A_m^T] \]

where \( A_i^T \) transforms the tip force of arm \( i \) to the force at the object frame \( C \), and it is given by

\[ A_i^T = \begin{bmatrix} I & r_{ic} \times \\ 0 & I \end{bmatrix} \]

where \( r_{ic} \) is the vector from the tip of arm \( i \) to the object frame \( C \) and \( \times \) denotes cross product in a coordinate representation. Clearly, \( A_i^T \) is nonsingular, and, hence, \( A_i^T \) is of full row rank. The force on the object at \( C \), \( A_i^Tf \), can be equivalently written as

\[ A_i^Tf = A_i^Tf_c = A_i^T\tilde{A}^Tf \]

(6)

where

\[ \tilde{A} = \begin{bmatrix} I \cdots I \\ 0 & A_m^T \end{bmatrix} \]

(7)

and \( f_c \) = \( \tilde{A}^Tf \). Note that \( \tilde{A} \) is square invertible since each \( A_i \) is nonsingular.

The matrix \( A_i^T \) is "fat", therefore, it possesses a nontrivial null space. \( f_c \) in this null space means that it does not contribute to the motion of the held object but only to the buildup of the internal force. Hence, we define the squeeze subspace to be \( X_s = \text{Ker}(A_i^T) \) (the kernel or the null space of \( A_i^T \)). The orthogonal complement
of $X_s$ is defined as the move subspace, which is $X_m = \text{Im}(A_e)$ (the image or the range space of $A_e$) since $R^m = X_m \oplus X_s$. For a given composite tip force, $f$, there exists a unique orthogonal decomposition for $f_e$:

$$f_e = f_{m} + f_{a},$$

where $f_{m} \in X_m$ and $f_{a} \in X_s$. We call this the move/squeeze decomposition. Only $f_{m}$ contributes to the motion of the held object. Clearly, $\text{dim}(X_s) = (m - 1) \cdot 6$ and $\text{dim}(X_m) = 6$. Associated with $X_m$ and $X_s$, we can define a pair of projection operators, $\mathcal{P}_m$ and $\mathcal{P}_s$, onto these subspaces. Since $A_e^T$ is full rank, we have

$$\begin{align*}
\mathcal{P}_m &= A_e (A_e^T A_e)^{-1} A_e, \\
\mathcal{P}_s &= I - A_e (A_e^T A_e)^{-1} A_e^T.
\end{align*}$$

(9) (10)

Note that $\mathcal{P}_m$ and $\mathcal{P}_s$ do not have the unit inconsistency problem noted in Duffy (1990) (there would be a problem if $A_e$ above is replaced by $A$).

A consequence of the move/squeeze decomposition is that any term in $\tau$ of the form $J^T \dot{A}^{-T} F_{c}$, with $F_{c}$ in the squeeze subspace, does not affect motion of the system when none of the arms is at a singular configuration. The motion of the arms, however, does affect the squeeze force, due to the squeeze component of the d'Alembert (inertial) force (i.e. the velocity dependent terms in $\mathcal{P}_s f_{a}$). This motivates the following decomposition of the control torque:

$$\tau = \tau_m + \tau_s + \tau_b,$$

(11)

where $\tau_m$ is responsible for the motion control, $\tau_s$ is responsible for the squeeze force control, and $\tau_b$ is a model-based feedforward compensation for the gravity load, and/or centrifugal and Coriolis forces, etc. The motion and force control problems can now be considered separately. Specifically, we propose the following general procedure. First, a stable motion controller is designed for $\tau_m$, with no consideration given to the internal (squeeze) force. Then design the squeeze force controller from $\tau_s = J^T \dot{A}^{-T} F_{c}$, $F_{c} \in X_s$, by regarding the motion-induced component of squeeze force as a disturbance that is unaffected by the squeeze force $f_{a}$ or the squeeze control $F_{c}$. The remainder of this paper concentrates on filling in the details of these steps.

3. MOTION CONTROL

3.1. Feedback linearization and arms-as-actuators approaches

Different choices of the control variable gives rise to different control architectures. We consider the following possibilities:

(1) A feedback linearization approach which uses the acceleration of a generalized coordinate.

(2) An arms-as-actuators approach which uses the tip force vectors of all arms.

(3) A full dynamics approach which uses the joint torque vectors of all arms.

In this subsection, we briefly discuss the first two approaches (a more detailed discussion is presented in Wen and Kreutz-Delgado (1991)), the last approach is discussed in the next subsection.

In the arms-as-actuator approach, the effective motion control problem involves the position and orientation of a rigid body (i.e. the held object). The arms are viewed as providing the "actuation signal" to the held object. This is done by using a state-dependent nonlinear feedforward that produces the desired contact forces. The rigid body dynamics of the held object seen at a frame fixed in the object are composed of two parts: a force balance equation (Newton's equation) and a torque balance equation (Euler's equation). The force balance equation only contains the gravity load and can be controlled easily. The torque balance equation involves the control on the rotation group, $SO(3)$. There has been a recent surge of interest in this problem—see Wen and Kreutz-Delgado (1990) and references therein. It has been shown that global asymptotic stabilization can be achieved by using the unit quaternion feedback.

In the feedback linearization approach, the effective motion control problem involves a system of decoupled double integrators using a generalized coordinate representation for the position/orientation of the held object. Hence, the control design problem is much simplified. However, the representation Jacobian (transforming angular velocity to a generalized velocity) in the control law introduces additional singularities which are a mathematical constraint rather than a physical limitation.

Since the multiple-arm system represents an overactuated system with respect to the payload, the redundant actuation can be used to control the squeeze force. This can be posed as an optimization problem, giving rise to the load-balancing problem as discussed in Kreutz and Lokshin (1988); Nakamura et al. (1987) and Alberts and Soloway (1988).

Full dynamical model information of the manipulator is needed in the nonlinear feedforward compensation for both the feedback linearization and arms-as-actuator schemes. Feedback linearization also requires held-object
model information. Furthermore, Jacobian non-singularity of all arms is required for all time. When the Jacobian is close to being singular, there would also be sensitivity problems. Furthermore, computational and robustness issues due to the complex nonlinear, model dependent compensation need to be addressed for a successful implementation. At the present, work in this direction is lacking in the context of multiple-arm systems. For this reason, the rest of the paper will focus on the full dynamics approach and develop relatively model-independent control laws directly for the joint torques.

3.2 The full dynamics approach

The inherent passivity property in mechanical systems such as robots has been much exploited by the robot control community starting from Takegaki and Arimoto (1981). The appeal of this approach is clear: only a small amount of model information (for gravity compensation) is needed to achieve globally asymptotic stable set point control. In this section, we will show the generalization of this method to the multiple-arm systems. Since the full nonlinear dynamics is considered in the control design problem with no explicit feedforward compensation, we call this the full dynamics approach.

The single arm analysis can be directly generalized to a multiple-arm system. The Lyapunov function candidate is chosen to be the total kinetic energy (for the arms and payload) and an artificial potential energy which has the global minimum at the set point. For simplicity, we consider a quadratic artificial potential energy only. There are three types of variables that can be used in the potential energy: (1) joint position (of all arms), (2) tip position (of all arms), (3) generalized coordinates. The first two over-specify the configuration of the system (due to the kinematic constraint imposed by the rigid grasp of a common object) and hence are not generalized coordinates. For generalized coordinates, there are many possible choices: position and orientation of the mass center of the held object, a subset of the tip position, joint position and/or tip forces. For the tip position and generalized coordinate cases, a parametrization for the arm tip and payload orientations need to be chosen. We will assume that a minimal representation is used, though other related works (Wen and Kreutz-Delgado, 1990) have indicated that the unit quaternion (Euler parameters) may be a better choice since there is then no problem with the singularity of representation.

For the set point control problem, consider

\[ V = \frac{1}{2}u^T M \dot{u} + \frac{1}{2} \dot{q}^T M \ddot{q} + U^* , \]

(12)

where \( U^* \) can be any one of the choices listed in Table 2. Note that in Table 2, \( \Delta x \triangleq x - x_{\text{des}} \) and \( \Delta x_c \triangleq x_c - x_{\text{set}} \) (\( x \) and \( x_c \) are as defined in Table 1 and the subscript ‘des’ denotes the desired quantity).

Motivated by the single-arm result (Takegaki and Arimoto, 1981), consider the following proportional-derivative-gravity control law (cf. (11)):

\[ \tau_m = -\tau_p - \tau_v, \quad \tau_v = k(q) - J^T F_c \]

(13)

where \( k \) is the arm gravity load, and \( F_c \) is gravity compensation for the held object, chosen to satisfy \( A^T F_c = k_c \). The proportional and velocity feedback terms, \( \tau_p \) and \( \tau_v \), are given in Table 3 and depend on the variables used for feedback. Note that \( J_i \) denotes the Jacobian matrix for the \( i \)th arm. The feedback gain \( D \) is chosen to ensure that all the internal degrees of freedom are damped when the arm is redundant or when the Jacobian becomes singular. The derivative of \( V \) along the solution then becomes

\[ \dot{V} = -\dot{q}^T K_c \dot{q}, \quad \dot{V} = -\dot{q}^T (D + J^T F_c) \dot{q}, \]

(14)

respectively, for the joint level, tip level, and generalized coordinate cases. By the Invariance Principle (LaSalle, 1960), (14) implies the asymptotic convergence of \( \dot{q}(t) \) to zero, i.e. the arms always reach a steady state configuration. This holds true even for unfeasible joint and tip set points (i.e. ones that do not satisfy the kinematic constraint) in the joint and tip level feedback cases. In the joint feedback case, the joint error and tip force converge to the

<table>
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<tr>
<th>Table 2. Quadratic Potential Energy Candidates</th>
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<tbody>
<tr>
<td>Choice of variable</td>
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<tr>
<td>Joint position error</td>
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<tr>
<td>Tip position error</td>
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<tr>
<td>Generalized coordinate error</td>
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<tr>
<th>Table 3. PD Feedback in Set Point Motion Control Laws</th>
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<tr>
<td>Type of feedback</td>
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<td>------------------</td>
</tr>
<tr>
<td>Joint level</td>
</tr>
<tr>
<td>Tip level</td>
</tr>
<tr>
<td>Generalized</td>
</tr>
<tr>
<td>Coordinate</td>
</tr>
</tbody>
</table>
manifold described by
\[ K_p \Delta q + J^T (F_k + f) = 0. \]  
(15)

In the tip feedback case, the tip error and tip force converge to the manifold described by
\[ J^T (K_p \Delta x + F_k + f) = 0. \]  
(16)

In the generalized coordinate feedback case, the generalized coordinate error and tip force converge to the manifold described by
\[ J^T F_p = 0, \quad A^T F_p = K_p \Delta x_c. \]

Even if the joint and tip set points are kinematically feasible, the above analysis still only implies convergence to these manifolds. For the joint and tip level feedback cases, they may contain a configuration where, because the multiple-arm kinematic constraint is not taken into account in the control law, the motion control component of each arm works against each other and contributes only to the buildup of the internal constraint force but not to any motion. For this reason, these manifolds are called the joint and tip Jam Manifolds, respectively. They will be discussed in more detail later. In the generalized coordinate feedback case, assuming the steady state Jacobian is nonsingular, the position error converges to zero since the kinematic constraint has been incorporated.

The fixed set point control laws discussed above provide a useful exercise as it demonstrates the use of a general class of Lyapunov functions that results in a number of simple stabilizing control laws and points out some interesting issues unique to multiple-arm control such as the choice of the feedback variable and the Jam Manifold. However, the fixed set point control paradigm is fundamentally flawed because the closed loop trajectory transient is not controlled. For initial condition far away from the desired set point, the transient is typically so wild that these control laws are virtually unusable. The problem is most severe in tip and generalized coordinate feedback, where arms may cross Jacobian singularities, flip poses (due to multiple solutions to the inverse kinematics problem), collide with themselves, violate joint stops, etc. An example, demonstrating the undesirable behavior of the set point control is shown in Section 6. This motivated us to extend our framework to include the trajectory tracking problem.

Motivated by the set point control case, first consider the following Lyapunov function
\[ V = \frac{1}{2} \Delta u_c^T M_c \Delta u_c + \frac{1}{2} \Delta q^T M \Delta q + U^*, \]  
(17)

where
\[ \Delta u_c = u_c - u_{\text{des}}, \]
\[ u_{\text{des}} = (A_T^T A_e)^{-1} A_T^T A^{-1} u_{\text{des}}, \]
\[ u_{\text{des}} = J(q) \dot{q}_{\text{des}}, \quad \Delta \dot{q} = \dot{q} - \dot{q}_{\text{des}}. \]

\( U^* \) is as given in Table 2. For the joint level feedback, only \( \dot{q}_{\text{des}} \) is needed. Therefore, even though \( u_{\text{des}} \) is related to \( \dot{q}_{\text{des}} \) through a Jacobian that is evaluated at \( q \) but not \( \dot{q}_{\text{des}} \) (to simplify the stability proof), the control law is not affected. For tip level and generalized coordinate level cases applied to a redundant arm, \( \dot{q}_{\text{des}} \) is needed. However, the fact that the integral of \( \dot{q}_{\text{des}} \) is not related to \( x_{\text{des}} \) and \( x_{\text{des}} \) by the forward kinematics has no consequence in the control implementation.

The derivative of the Lyapunov function candidate along the solution can be computed as
\[ \dot{V} = \Delta \dot{q}^T \tau + \dot{U}^* - \Delta \dot{v}_c^T M_c \Delta v_{\text{des}} - \dot{\Delta \dot{q}}^T M \Delta \dot{q}_{\text{des}} - v_{\text{des}}^T \dot{b}_c + \dot{q}_{\text{des}}^T \left( (-C(q, \dot{q}) + \frac{1}{2} \dot{M}(q, \dot{q})) \dot{q} \right). \]

(19)

Now substitute in the PD plus gravity compensation motion control law and Table 3 (with \( \dot{q}, v \) and \( u_c \) in Table 3 changed to \( \Delta \dot{q}, \Delta v \) and \( \Delta u_c \), respectively). Then the solution of \( V \) along the solution can be bounded by
\[ \dot{V} \leq -\lambda \| \Delta \dot{q} \|^2 + \eta_1(t) \| \Delta \dot{q} \|^2 + \eta_2(t) \| \Delta \dot{q} \|, \]

(20)

where \( \eta_1 \) and \( \eta_2 \) are terms dependent on the model parameters and the desired trajectory.

Consider desired trajectories that belong to
\[ S = \{ \dot{q}_{\text{des}}(t) \rightarrow 0, \text{ as } t \rightarrow \infty, \max \{ \dot{q}_{\text{des}}, \dot{q}_{\text{des}} \} \in L_2([0, \infty)), \dot{q}_{\text{des}} \text{ is uniformly continuous} \}. \]

These are mild restrictions in the case of point-to-point operation (with transient shaping), i.e. the desired trajectories reach a steady state. Therefore, we call \( S \) the class of "moving set points". More general classes of trajectories (e.g. periodic) can be considered by using the stability argument in Ortega and Spong (1988); this will be discussed in a future communication.

For a desired trajectory in class \( S \), \( \eta_1(t) \rightarrow 0 \) as \( t \rightarrow \infty \), and \( \eta_2(t) \in L_2([0, \infty)) \). By integrating both sides of (20) from \( t_0 \) to \( t \), where \( t_0 \) is sufficiently large so that \( |\eta_1(t)| < \lambda \) for all \( t \geq t_0 \), there exists \( \lambda_1 > 0 \) such that
\[ \lambda_1 \| \Delta \dot{q} \|_{L_2([t_0, t])} \]
\[ \leq V(t_0) - V(t) + \int_{t_0}^{t} \eta_2(\tau) \| \Delta \dot{q}(\tau) \| \, d\tau \]
\[ \leq V(t_0) + \| \eta_2 \|_{L_2([t_0, t])} \| \Delta \dot{q} \|_{L_2([t_0, t])}. \]

(21)
TABLE 4. STABILITY PROPERTIES OF MOVING SET POINT MOTION CONTROLlers

<table>
<thead>
<tr>
<th>Type of feedback</th>
<th>Stability property</th>
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<tbody>
<tr>
<td>Joint level</td>
<td>$\dot{\theta} \rightarrow 0$, $\Delta \theta$ converges to the manifold $K_p \Delta \theta + J^T(K_p \Delta x + f) = 0$.</td>
</tr>
<tr>
<td>Tip level</td>
<td>$v \rightarrow 0$, $\Delta \alpha$ converges to the manifold $J^T(K_p \Delta x + f) = 0$.</td>
</tr>
<tr>
<td>Generalized coordinate</td>
<td>$v_c \rightarrow 0$, $\Delta \alpha_c$ converges to the manifold $(J^Tf = 0, A^Tf = K_p \Delta \alpha)$.</td>
</tr>
</tbody>
</table>

Now, by completing the squares involving $||\Delta \dot{q}||_{L_2([t_0, \infty))}$, it follows that $\Delta \dot{q} \in L_2([t_0, \infty))$. From (21), $V(t)$ is uniformly bounded for all $t$, which implies $\Delta q$, $\Delta \dot{q}$, $\Delta \ddot{q}$ are uniformly bounded. From the dynamical equation, $\ddot{q}$ is also uniformly bounded which implies that $\dot{q}$ is uniformly continuous. Again by using the dynamical equation, $\ddot{q}$ is also uniform continuous. Since $q_{des}$ is assumed to be uniform continuous, so is $\Delta \ddot{q}$. We have shown that $\Delta \ddot{q} \in L_2$ and it is uniformly continuous, therefore, $\Delta \ddot{q} \rightarrow 0$ as $t \rightarrow \infty$. Furthermore, the uniform continuity of $\Delta \ddot{q}$ implies that $\Delta \ddot{q} \rightarrow 0$. It follows from the arm dynamical equation that $\tau_p + J^Tf = 0$ which yields the same convergence result as in the fixed set point case.

The stability properties of the “moving set point controllers” can now be summarized below which includes the set point case.

Result 1. If the desired trajectory belongs to class $\mathcal{F}$, then the multiple-arm system with control laws (13) and Table 3 has the stability property shown in Table 4. Furthermore, if the initial tracking error is zero, the maximum trajectory tracking error is inversely proportional to the size of the PD gains.

For the generalized coordinate feedback case, it is certainly true that the Jam Manifold is guaranteed to be just the zero configuration. For the other two cases, it may be that the nonzero error jammed configuration exists. However, the attractivity of these configurations remains a topic for future research. It is valid, then, to ask why joint feedback and tip feedback are considered at all. There are several answers to this question. Low level joint level servo is a fact of life in most industrial robots. It is reassuring to know that these arms can work stably together even if the set point joint angles do not satisfy the kinematic constraint precisely. The absence of the Jacobian weighting in the control law in the joint feedback case also implies tracking performance independent of the near singularity of the Jacobian. The configuration space (joint space) may also be the natural domain of specification for some obstacles (e.g., joint stops). The tip coordinate feedback may be necessary if task specification is with respect to the end effector locations and the held object kinematics is not precisely known.

If the desired trajectory starts with the same initial condition as the actual trajectory and has the desirable transient behavior, such as, small overshoot, no excessive acceleration or jerk, avoidance of Jacobian singularities, joint stops, obstacles and the arms themselves; then Result 1 shows that high enough feedback gains ensure that the actual trajectory will have similar properties, also. The maximum tracking error can be shown to be proportional to the $L_2$-norm of $\eta_2$ in $V$ which is composed of the difference between the moving set point and its steady state, the desired velocity and acceleration. A trajectory planning problem can be posed to find a desired trajectory that satisfies the required transient response and minimizes the $L_2$-norm of $\eta_2$.

Control laws that incorporate the full model information can also be constructed within this approach (by using, for example, results in Wen and Bayard (1987)) for the tracking of more general classes of trajectories (such as periodic motion). We can qualitatively state the advantage of this added complexity in the control algorithm. In the tracking control problem, even if the initial tracking error is zero, the PD control law will always incur a nonzero trajectory tracking error. This error can be made small if gains are allowed to be large; however, it may not always be practical, given the limited actuator size and the noise problem. With the model-dependent control laws, the tracking error will remain zero (at least theoretically; noise and model error will cause some deviation from the desired trajectory). The same is true in the internal force control (see next section). If full model information is assumed, precise force control at every moment in time is possible; while the model-independent control law reduces finite force error only with the use of high gains. The type of control law to use for a given application depends on the trade-off between the available a priori model information and the performance requirement, subject to actuator and sampling constraints.

It has been observed in simulation that tracking performance deteriorates in the tip variable and generalized coordinate feedback cases when the Jacobian is close to being singular. One way to compensate for this effect is to adapt the gains $K_p$ and $K_v$. Suppose $V$ in
has terms added of the form \( \frac{1}{2} \text{tr} [K_p^T \Lambda_p^{-1} K_p] + \frac{1}{2} \text{tr} [K_v^T \Lambda_v^{-1} K_v] \), the control laws in Table 3 now include the terms \(-\dot{K}_p \Delta r - \dot{K}_v \Delta \xi\) where \((r, \xi)\) correspond to either \((q, \dot{q})\), \((x, v)\) or \((x_v, v_v)\), depending on the type of feedback variables, \(K_p\) and \(K_v\) are adaptively updated via the standard gradient scheme with leakage (to prevent windup due to noise)

\[
\dot{K}_p = \Lambda_p (\Delta r \Delta r^T - \sigma_p \dot{K}_p),
\]

\[
\dot{K}_v = \Lambda_v (\Delta \xi \Delta \xi^T - \sigma_v \dot{K}_v),
\]

where \(\Lambda_p\) and \(\Lambda_v\) are positive definite matrices controlling the rate of adaptation. Then the same bound of \(\dot{V}\) holds as in (20). Hence, the same closed loop stability property as in Result 1 follows. However, the feedback gains will self regulate based on the size of the tracking error. The coefficients \(\sigma_p\) and \(\sigma_v\) (so-called leakage parameters in the adaptive control literature) prevents the saturation of \(\dot{K}_p\) and \(\dot{K}_v\) when a persistent noise is present.

4. SQUEEZE FORCE CONTROL

In this section, we consider the problem of choosing \(\tau_c\) in (11). To avoid affecting the motion control, \(\tau_c\) is chosen to be the following form:

\[
\tau_c = J^T \dot{A}_c F_c = J^T \dot{A}_c \xi,
\]

where \(\dot{A}_c^T\) is a full rank matrix whose columns span the null space of \(A_c^T\) (i.e. \(\text{Im} \dot{A}_c^T = X_c\)). The effective control variable for force control is now \(\xi\). Now, the squeeze projection of the contact force in (5) can be written as

\[
f_{c_a} = F_c + \eta = \dot{A}_c \xi + \eta,
\]

where \(\eta\) is the motion-induced squeeze force and it tends to zero as \(t \to \infty\) if an asymptotically stable motion control law is used. The effective squeeze control variable \(\xi\) can be selected in two ways. One method poses an optimization problem (usually called the load balancing problem [Kreutz and Lokshin, 1988; Nakamura et al., 1987]) and the other treats it as a set point control problem. Due to the similarity to Kreutz and Lokshin (1988) and Nakamura et al. (1987), the optimization approach will only be addressed briefly. The set point control approach will be treated in greater detail.

It is reasonable to choose \(\xi\) at each point in time through the following minimization problem:

\[
\min_{\xi} \left[ \tau^T Q_1 \tau + (f_{c_a} - f_{c_{\text{aim}}})^T Q_2 (f_{c_a} - f_{c_{\text{aim}}}) \right]
\]

\[
= \min_{\xi} \left[ (\tau_m + \tau_\xi + D\xi)^T Q_1 (\tau_m + \tau_\xi + D\xi) \right]
\]

\[
+ (\dot{A}_c \xi + \eta - f_{c_{\text{aim}}})^T Q_2 (\dot{A}_c \xi + \eta - f_{c_{\text{aim}}}),
\]

where \(D \equiv J^T \dot{A}_c^T\) and \(Q_1, Q_2\) are arbitrary positive definite operators. The first term in the optimization index is motivated by power consumption considerations (Kreutz and Lokshin, 1988) and the second term is to ensure that the squeeze force does not deviate too far from a certain desired value, \(f_{c_{\text{aim}}}\). There are usually other hard constraints such as upper and lower bounds on \(f_{c_a}\) and \(\tau\). This constrained minimization problem can be solved in principle (via quadratic programming, such as in Nakamura et al. (1987)) but will not be further explored here.

If no constraint is imposed, the problem can be easily solved analytically:

\[
\xi^* = -(D^T Q_1 + \dot{A}_c^T Q_2 + \dot{A}_c^T Q_2 (f_{c_{\text{aim}}}) - \eta)^{-1}
\]

\[
\times (D^T Q_1 (\tau_m + \tau_\xi + D\xi) + \dot{A}_c^T Q_2 (f_{c_{\text{aim}}}) - \eta)).
\]

where \(\dot{A}_c \equiv (\dot{A}_c^T)\) is the annihilator of \(A_c\). Note that the inverse in the above expression always exists (independent of the Jacobian singularities) since \(\text{Ker}(\dot{A}_c^T) = \{0\}\) by construction. The \(\eta\) term in (26) requires the complete model information to implement. If a good motion control strategy is used, a reasonable suboptimal solution will be obtained by setting \(\eta = 0\) in (26). Then the solution is “asymptotically optimal” since \(\eta \to 0\).

If it is desired to have tight control of the squeeze force, such as in the manipulation of delicate objects, it may be necessary to choose \(\xi\) from a set point squeeze control problem. However, without using full model information, the squeeze force can only be controlled asymptotically. We will show that if force feedback is used, tight control of squeeze force can still be achieved without much model information. In general, either a feedforward or feedback control structure may be used to drive the squeeze force to its set point asymptotically, depending on whether force sensors are available. In order to use the full composite force vector in the feedback control law, each arm needs to be equipped with a force sensor. The resulting feedback strategy, if properly applied, should have better performance and robustness than the feedforward strategy. Furthermore, if the gravity load in the squeeze subspace is not fully compensated, feedback control can still result in zero squeeze error.
while the feedforward control will incur a squeeze force error.

As described earlier, an important property of $\eta$ in (25) (due to the move/squeeze decomposition) is that it is not affected by $F_a$. Hence, it can be treated as an external disturbance. We assume that a stable motion control strategy has been used, so that $\eta(t) \to 0$ as $t \to \infty$. We are interested in studying the following aspects of the force control problem.

(1) Stability. (Does $f_a(t) \to f_{a_{\text{res}}}$ as $t \to \infty$ in (25)?)

(2) Transient performance. (What is the maximum force error, i.e. $\max_{t \geq 0} \|f_a(t) - f_{a_{\text{res}}}(t)\|$?)

(3) Convergence rate. (How fast does $f_a \to f_{a_{\text{res}}}$?)

(4) Noise reduction. (If $\eta(t) \to \eta_\infty \neq 0$, representing a persistent noise, what is the steady state force error?)

(5) Robustness with respect to time delay in force measurements. (If a force feedback strategy is used, how does time delay in the force measurement affect stability?)

To attain asymptotic stability equation (25), a feedforward control will suffice

$$F_a = f_{a_{\text{res}}}.$$  (27)

However, the transient performance and convergence rate are determined entirely by $\eta$ (which are in turn determined by the quality of the motion control law). There is also no noise reduction in this scheme.

If the arm tip forces are measured, then clearly a feedback strategy is preferable, due to the hope for added insensitivity to noise and improved transient performance. However, the infinite rigidity assumption stated in Section 2 necessitates extra care in the control design. The lack of dynamics in (25) means that finite bandwidth feedback from $f_a$ to $F_a$ would violate the strict causality of the loop and this has some unintended consequences. For example, the control law (which we do not recommend)

$$F_a = f_{a_{\text{res}}} + \beta(f_a - f_{a_{\text{res}}}),$$  (28)

normally implies $f_a \to f_{a_{\text{res}}}$ for $\beta \neq 1$. Furthermore, transient performance, convergence rate and steady state error due to noise can all be much improved over the feedforward case, if $\beta$ is large. However, an arbitrarily small time delay in the feedback channel (which is always present in a physical implementation) leads to instability if $|\beta| > 1$. If $|\beta| < 1$, then the response of the resulting linear discrete time system consists of two terms: the homogeneous solution and the particular solution due to $\eta$. For fast convergence of the homogeneous solution to zero, $\beta$ needs to be close to zero, but then the response is similar to that of the feedforward control and the desirable properties due to the force feedback is lost.

Recognizing that the problem is caused by the algebraic loop due to the proportional force feedback, we suggest pre-processing the measured force by a strictly causal filter (if the filter is linear, then strictly proper). The feedback control law then takes on the following form:

$$F_a = f_{a_{\text{res}}} + \mathcal{I}(f_a - f_{a_{\text{res}}} ),$$  (29)

where $\mathcal{I}$ is a strictly proper linear filter such that $(I - \mathcal{I}) \text{ has zeros only in the open left half plane.}$ Clearly, $f_a(t) \to f_{a_{\text{res}}}$ as $t \to \infty$. To see the transient behavior, we note from (25) and (29) that

$$\Delta f_a = f_a - f_{a_{\text{res}}} = \mathcal{L} \ast \eta,$$  (30)

where $\ast$ denotes the convolution operator and $\mathcal{L}$ the convolution kernel associated with $(I - C)^{-1}$. Since $\mathcal{L}$ can contain arbitrarily fast dynamics (if the desired dynamics of $\mathcal{L}$ is $a(s)/b(s)$ in the Laplace domain) then

$$C(s) = \frac{a(s) - b(s)}{a(s)},$$

is the Laplace transform of the corresponding filter $\mathcal{I}$, the $L_1$ norm of $\mathcal{L}$ can be made arbitrarily small. By the following error estimate (Desoer and Vidyasagar, 1975, Appendix C),

$$\|\Delta f_a\| = \|\mathcal{L}\|_{L_1} \|\eta\|_{L_\infty},$$

it follows that the transient performance and convergence rate can both be improved. To see the effect of a DC bias (due to, e.g. lack of gravity compensation), assume $\lim_{t \to \infty} \eta(t) = \eta_\infty \neq 0$.

From the final value theorem, $\lim_{s \to \infty} s \eta(s) = \eta_\infty$, or, $\eta(s) \approx \eta_\infty / s$, for $s$ close to zero. This implies $\Delta f_a(s) \approx L(s) \eta_\infty$, where $L(s)$ is the Laplace transform of $\mathcal{L}$. Hence,

$$\lim_{s \to \infty} \Delta f_a(s) = \lim_{s \to \infty} s \Delta f_a(s)$$

$$= \lim_{s \to \infty} L(s) \eta_\infty$$

$$= \lim_{s \to \infty} \frac{a(s)}{b(s)} \eta_\infty.$$

We conclude that if $C(s)$ has a pole at the origin, i.e. $\lim_{s \to 0} a(s) = 0$, then there is no steady state error. To summarize: all the control objectives are satisfied with the control law (29), provided that $(I - C)^{-1}$ is a stable filter and $C$ has a pole at the origin. If the spectrum of $\eta(t)$ is known (say, for a repeated cyclical task), $C$ can be
chosen to selectively notch out the dominant dynamics in \( \eta(t) \). What about robustness with respect to small time delays? To address this problem, we use a first order approximation of \( f_c(t - \Delta t) \), for \( \Delta t \) small, i.e.

\[
f_c(t - \Delta t) \approx f_c(t) - f_c(t) \Delta t.
\]

The closed loop system in the Laplace domain now becomes

\[
\Delta f_c(s) = (I - C(s) - \Delta t C(s)s)^{-1} \eta(s). \tag{31}
\]

Since \( C(s) \) is strictly proper and \( (I - C)^{-1} \) is stable, for sufficiently small \( \Delta t \), the perturbed system (31) remains stable. In the case of direct proportional feedback, \( C \) is not strictly proper and indeed the corresponding closed loop system becomes unstable for arbitrarily small \( \Delta t \).

A particularly simple choice of \( C \) is just an integrator, i.e. in the Laplace domain,

\[
C = -\frac{\beta}{s}. \tag{32}
\]

This control law has all the desirable features discussed above. If the integral feedback gain \( \beta \) is chosen sufficiently large, and \( \eta(t) \) is uniformly bounded in \( t \), then by explicitly solving the closed loop dynamical equation, it can be shown that the transient effect of \( \eta \) on \( f_c - f_{c_{\text{des}}} \) can be made arbitrarily small.

The discussion on set point squeeze force control can be summarized in the result below:

**Result 2.** For the multiple-arm control system under consideration, if the arm configuration converges to a steady state (i.e. velocity converges to zero) and the gravity load is fully compensated, then either the feedforward controller (28) or the feedback controller (29) with \( C \) a strictly proper linear filter and \( (I - C) \) containing zeros only in the open left half plane, drives \( f_c \to f_{c_{\text{des}}} \).

If in (29), \( C \) has a pole at the origin, then replacing \( \tau_i \) in (11) by its projection in the move subspace (i.e. if no gravity compensation in the squeeze subspace is done) does not affect the asymptotic convergence of \( f_c - f_{c_{\text{des}}} \) to zero, and in general, \( f_c \to f_{c_{\text{des}}} \) even if \( \eta \to \eta_{\text{ref}} \neq 0 \).

If \( C \) is chosen to be an integrator as in (32) and \( \eta \) in (31) is uniformly bounded in time then \( f_c(t) - f_{c_{\text{des}}} \) tends to zero uniformly for \( t \) in bounded intervals as \( \beta \to \infty \).

In practice, the closed kinematic chain would always have some flexibility (e.g. joint flexibility, force/torque sensor flexibility, etc.). As shown in Wen and Murphy (1991), the product between the integral gain, \( \beta \), and the stiffness associated with the flexible component must therefore be chosen sufficiently small to ensure stability.

5. SOME COMMENTS ABOUT JAMMING

Under non-generalized-coordinate feedback (such as joint level and tip level control laws), we showed that the arms converge to the Jam Manifold in Section 3. The members in the Jam Manifold other than the desired set point are called the jammed configurations, since they represent balance of all the internal forces due to the applied torque and gravity. In this section, we consider the case that the set point \( (q_{\text{des}} \text{ or } x_{\text{des}}) \) is kinematically feasible (i.e. satisfies the multiple arm constraint). We will derive a sufficient condition that guarantees the absence of any jammed configuration for nonredundant arms. A simple three-link example will also be analyzed to show that jammed configurations can occur in some situations.

Consider first the joint level control. The asymptotic manifold is of the form:

\[
\sum_{i=1}^{m} A_i^T J_i^{-T} K_{p_i} \Delta q_i = 0. \tag{33}
\]

Let \( \mathcal{L}_i \) denote the forward kinematics transform for the arm \( i \) and \( \mathcal{L}_n \) the kinematic transformation between tip position and orientation of arm \( j \) to the tip position and orientation of arm \( k \). Then (33) can be written as

\[
\Delta q_i + \sum_{i=1}^{m} K^{-1}_{p_i} J_i^T \cdot A_i^T \mathcal{L}_i(q_i) \mathcal{L}_n^{-1}\mathcal{L}_i(q_i)) = 0. \tag{34}
\]

Let \( \mathcal{L}_n^{-1} \) denote the inverse kinematics transformation from the tip position and orientation to the joint angles of arm \( i \), with the range appropriately restricted so that it is a single valued map and the arm is away from its singularities. Then from the kinematic constraint imposed by the nonredundant arms rigidly grasping the object, \( q_i, i \neq j \), can be determined from \( q_j \) as follows:

\[
q_i = \mathcal{L}_n^{-1}(\mathcal{L}_n(q_i)).
\]

Hence, (35) can be stated as

\[
q_i = \mathcal{S}_i(q_j), \tag{35}
\]

where \( \mathcal{S}_i \) is some nonlinear map: \( \mathbb{R}^n \to \mathbb{R}^n \) with a fixed point at \( q_{\text{des}} \). Noting that \( J_i \) and \( A_i \) have bounded derivatives and incremental change of change of \( q_j \) causes finite incremental change in \( q_i, \ i \neq j \) (when \( q \) is restricted to be in a suitable bounded region), it can be argued that

\[
\left| \frac{d\mathcal{S}_i}{dq_j} \right| \leq \alpha \Pi_{i,j} \sigma_{\text{max}}(K_{p_i}), \tag{36}
\]

where \( \sigma_{\text{max}}(K_{p_i}) \) is the maximum singular value of \( K_{p_i} \).
for some constant α which depends on \( q_{\text{des}} \) (which is a constant vector for the set point case considered here). Note that \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) denote maximum and minimum singular values, respectively. Hence, if \( \frac{\Pi_{i} \sigma_{\text{max}}(K_{p})}{\sigma_{\text{min}}(K_{p})} \) is sufficiently small (relative to α) for any \( j \in \{1, \ldots, N\} \), \( S_{j} \) is a contraction and \( q_{\text{des}_j} \) is the only fixed point of (35). Therefore, in this case, jamming cannot occur. For the general situation, we conjecture that \( S_{j} \) consists of finite number of fixed points, with the number depending on the \( K_{p} \) and the arm kinematic parameters.

The tip level feedback case is somewhat easier to analyze due to the lack of a Jacobian term in the manifold condition. Write the manifold condition as follows:

\[
\sum_{i=1}^{m} A_{i}^{T} K_{p_i} \Delta x_i = 0. \tag{37}
\]

For simplicity let \( K_{p_i} \) be scalars denoted by \( k_{p_i} \). Now, \( A_{i}^{T} \) can be written as

\[
A_{i}^{T} = \begin{bmatrix} I & r_{i} \times \end{bmatrix}, \tag{38}
\]

where \( r_{i} \) is the vector from the tip of \( i \)th arm to the origin of a coordinate frame that is invariant with respect to the object coordinate frame, and \( r_{i} \times \) denotes the cross product operation. Let \( \Delta x_i \) be

\[
\Delta x_i = \begin{bmatrix} \Delta p_i \\ \Delta z_i \end{bmatrix}, \tag{39}
\]

where \( p_i \) is some minimal representation of the tip orientation of arm \( i \) and \( z_i \) is the tip position of arm \( i \) and Δ denotes the difference between actual and desired. Choose the point \( c \) to coincide with the tip of the \( j \)th arm. Then (37) can be written as

\[
\sum_{i=1}^{m} k_{p_i} \Delta p_i + \sum_{i=1}^{m} k_{p_i} r_{ij} \times \Delta z_i = 0, \tag{40}
\]

\[
\sum_{i=1}^{m} k_{p_i} \Delta z_i = 0. \tag{41}
\]

For each \( i \),

\[
\Delta z_i = \Delta z_i + \Delta r_{ij}
\]

where \( \Delta r_{ij} = r_{p_i} - r_{\text{des}_j} \). Since we are considering the case that the desired set point is kinematically feasible, \( r_{\text{des}_j} \) is simply \( r_{ij} \) rotated from the \( j \)th robot tip frame to the desired \( j \)th frame. Hence, when there is no orientation error (i.e. the actual and the desired \( j \)th tip frame coincide), \( \Delta r_{ij} = 0 \).

It then follows from (41) that

\[
\Delta z_i = -\left( \sum_{i=1}^{m} k_{p_i} \right)^{-1} \left( \sum_{i=1}^{m} k_{p_i} \Delta r_{ij} \right). \tag{42}
\]

Substituting this expression into (40), then

\[
\sum_{i=1}^{m} k_{p_i} \Delta p_i = \left( \sum_{i=1}^{m} k_{p_i} \right)^{-1} \cdot \left( \sum_{i=1}^{m} \sum_{i=1}^{m} k_{p_i} r_{ij} \times \Delta r_{ij} \right). \tag{43}
\]

Note that (43) completely specifies the Jam Manifold since \( \Delta r_{ij} \) only depends on the orientation error.

To obtain further insight about the Jam Manifold, let us discuss two special cases.

- If the orientation error is zero for the end effector coordinate of any arm, then we can consider that arm as the \( j \)th arm in the above analysis. Since \( \Delta r_{ij} \) in (42) and (43) are all zeros in the \( j \)th tip frame, \( \Delta z_i = 0 \) and \( \Delta p_i = 0 \), for all \( i \). In other words, jamming does not occur with just position error alone.

- Consider the two planar 3 DOF arms as in Section 6. In this case,

\[
\Delta p_1 = \Delta p_2 = \Delta \theta, \tag{44}
\]

\[
r_{12} = l \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}, \tag{45}
\]

\[
\hat{r}_{12} = l [\sin(\theta); -\cos(\theta)], \tag{46}
\]

where \( l \) is the length of the bar held by the two arms. Equations (42)–(43) can now be stated as

\[
\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \frac{k_{p_1}}{k_{p_1} + k_{p_2}} l^2 \begin{bmatrix} \cos(\theta) - \cos(\theta_d) \\ \sin(\theta) - \sin(\theta_d) \end{bmatrix}, \tag{47}
\]

\[
\Delta \theta = -\frac{k_{p_1} k_{p_2}}{(k_{p_1} + k_{p_2})^2} l^2 \sin(\Delta \theta). \tag{48}
\]

A necessary and sufficient condition for a non-zero solution of (48) to exist is that

\[
\frac{k_{p_1} k_{p_2}}{(k_{p_1} + k_{p_2})^2} l^2 > 1. \tag{49}
\]

This condition will be violated if the ratio between \( k_{p_1} \) and \( k_{p_2} \) or between \( k_{p_1} \) and \( k_{p_2} \) is sufficiently large. Recall the same sufficient condition for the joint feedback case exists also. By looking at the intersection between curves defined by the left hand side and the right hand side of the \( \Delta \theta \) equation in (48), it is evident that even if (49) is satisfied, there can only be a finite number of jammed positions. The attractivity of these equilibria for the
closed loop system remains a topic for future research.

6. SIMULATION RESULTS
A system of two three-degree of freedom planar arms holding a rigid bar, as shown in Fig. 1, is simulated with the closed loop controllers discussed in this paper. The symbolic form of (1)–(4) is derived and used in a fourth-order variable step size Runge–Kutta ODE solver. Two different methods are used to solve the ODE. The first method integrates the ODE for the dynamics of the rigid bar at the object frame C. The second method integrates the joint equation with the algebraic kinematic constraint due to the closed chain imposed. The second method is used predominantly even though more integration variables are involved than the first method. The reason is that the inverse kinematics needed for the first method may yield discontinuous joint trajectories.

The arm geometric and mass parameters are selected to emulate the PUMA 560 arm moving in a plane (i.e. joints 1, 4 and 6 are locked). The mass values and inertia values are with respect to the link centers of mass. Numerical values of the parameters can be found in Wen and Kreutz-Delgado (1991).

Orientation of the object frame C is defined as the counterclockwise angle from the x-axis to the rigid bar. Orientation of the arm tip is defined as the sum of all the joint angles, which also corresponds to the counterclockwise angle from the x-axis to the third link. The orientation of the left arm is the same as the orientation of the object and the orientation of the right arm is the orientation of the object added with π. The left arm is initially in elbow down pose and right arm in elbow up pose. The initial configuration is shown in Fig. 1.

We will present simulation results from the following two cases:

1. Performance comparison between fixed and moving set point control laws.
2. Integral feedback squeeze control vs feedforward squeeze control.

The initial condition object frame C position is chosen to be (orientation = 0 rad, x = 0.3 m, y = 0.7 m). The object frame C position (orientation = π/2 rad, x = 0.4 m, y = 0.2 m) is used as the set point. For the joint level control, elbow up pose is chosen for the left arm and elbow down for the right arm.

6.1. PD set point control with and without transient shaping
The desired trajectory is generated by using the following formula:

\[
\begin{align*}
\theta(t) &= \theta_f - (\theta_f - \theta_i)e^{-at}, \\
\dot{\theta}(t) &= 2t\alpha(\theta_f - \theta_i)e^{-at},
\end{align*}
\]

where \( \theta \) is either \( q_{\text{des}}, x_{\text{des}}, \) or \( x_{\text{curr}} \), depending on the type of feedback used. The subscript \( f \) stands for the final set point and the subscript \( i \) stands for initial condition. The speed of convergence is controlled by the parameter \( \alpha \).

Plots of the object frame C orientation and x–y positions under joint, tip and object frame C feedback are shown in Figs 2–4 for the fixed set point case, and Figs 5–7 for the moving set point case.

Comparison between the transient shaping and no transient shaping cases shows a dramatic improvement in the transient response when transient shaping is used. The difference is most striking in the tip and object frame feedback cases, where if fixed set point control is used, both arms flipped poses, collided with themselves or each other and crosses Jacobian
singularities. All of these problems are avoided when transient shaping is used. Since motion affects squeeze force (due to the d'Alembert force), large motion transient also adversely affects the squeeze force profile.

6.2. Feedback squeeze control

As discussed in Section 4, we expect the force feedback control to result in a smoother transient response. This is confirmed in the simulation in this section together with the object frame position feedback control.

The following orthonormal basis for the squeeze subspace is chosen:

\[ f_{s_1} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad f_{s_2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \cos \theta \\ \sin \theta \end{bmatrix} \]

\[ f_{s_3} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ -\sin \theta \\ \cos \theta \end{bmatrix} \]
where $\theta$ is the orientation of the held object. The first basis represents a pure torque balance (balanced twist). The second basis represents an axial force balance (balanced squeeze). The third basis represents a shear force balance. The squeeze force set point is selected to be 5 N pure axial squeeze force:

$$f_{\text{core}} = 0 \cdot f_{s_1} + 5 \cdot f_{s_2} + 0 \cdot f_{s_3}.$$  

The feedback squeeze control is of the form

$$f_s = f_{\text{core}} - \beta \int_{t_0}^{t} (f_s(s) - f_{\text{core}}) \, ds.$$

Magnitudes of the three squeeze force components under feedforward and feedback control are shown in Figs 8–10, for $\beta = 0, 5, 50$, respectively. The gravity load in the squeeze subspace is not compensated, so a steady state error can be seen in the feedforward case. In general, feedback force control yields smaller overshoot, much better transient response, and no steady state error (due to the integral action). For a fixed tracking trajectory, it may be possible to use a more complex filter, instead of an integrator, to "notch" out the effect of transient motion in the squeeze subspace.

7. CONCLUSION

A general paradigm is presented in this paper for the control of multiple manipulators holding a rigid object. The rigidity assumption on the held object allows a useful decomposition of the contact spatial force into a component moving the held object and another component contributing only to the internal forces. Consequently, the motion and force control problems can be independently analyzed. In the motion control, depending on the choice of variables used in the control design, one obtains three different control structures: the full dynamics, arms-as-actuator and feedback linearization control structures. These structures demonstrate a trade-off between the complexity of the model used in the control design vs the complexity and the model dependency of the feedforward compensation. In the full dynamics approach, by using an energy Lyapunov function, a class of relatively model independent stabilizing control laws is developed. The actuator redundancy in a multiple-arm control system can be resolved either through an optimization (load balancing) or a squeeze force set point control problem. In the latter, we showed that integral force feedback control can achieve good performance and possesses good robustness properties. Generalization to related situations, such as an object in contact with the environment, multiple degrees of freedom contacts (point contact, finger, etc.), partial kinematic constraints (object with internal degrees of freedom such as scissors) and mobile base, is possible and will be communicated in the future. Future research topics include a flexible held object and/or a
flexible environment (in the contact case) and robustness analysis with respect to the grasp map $A^7$.

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