Discrete Implementation and Adaptation of Sliding Mode Control for Robot Manipulators

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Abstract
Two phenomena usually limit the accuracy in standard robot position control: joint friction and sensor noise. The high gain nature of sliding control offers promise to reduce the positioning error due to friction, but sensor noise can cause an unacceptable level of chattering. A frequent solution is to add a boundary layer. We show by both theoretical argument and experimentation, that sliding control with a boundary layer does not outperform proportional–derivative (PD) control due to the fact that the gain in the boundary layer cannot exceed the PD gain, due to noise and discretization. In this paper, we propose a combined switching and PD control with the level of switching selected so the position error is reduced but slightly higher than the noise level so that chattering will not occur. The switching level can be further adaptively updated based on the position error level. Through extensive experimentation of a six-DOF arm, we demonstrate that the proposed controllers avoid chattering and reduce the transient and steady state position errors of the PD control alone.

1 Introduction
Sliding Mode Control (SMC) is an outgrowth of relay control and bang-bang control, which, because of its simple realization and effectiveness, was among one of the first control applications in the history of automatic control theory. Extensive theoretical studies by V. Utkin [1] and U. Itkis [2] on systems with sliding modes appeared as early as in the seventies. They have analyzed systems with sliding modes from the mathematical point of view and provided a framework for synthesis, analysis and notation of SMC.
The first attempt to actually apply SMC to a robot was made by Young [3] in 1978. His motivation was that model based algorithms like the Computed Torque Control (CTC) require detailed model information of the manipulator. For SMC it is sufficient just to know the bounds of the model parameters. Through a two-link simulation, Young found the SMC to be insensitive to parameter variations and provided decoupling between joint motion.

A serious drawback to the successful implementation of the sliding mode control is the problem of chattering, where the control signal oscillates between extremes. This phenomenon is highly undesirable as it may excite the flexible modes of the robot and cause structural fatigue or failure. The equivalent control approach was introduced in [4, 5] where the computed torque component is augmented with a switching component to reduce the amount of chattering. To further remove the chattering, a boundary layer can be used in place of the sliding surface [6]. Essentially, the switching function is approximated by a continuous saturation function. There have been additional modifications by adaptively adjusting the width of the boundary region in [7].

The goal of this paper is to report our results on the experimental implementation of the sliding mode controller on a PUMA 560 arm. In contrast to the experimental work reported elsewhere in the literature, we found that for our experimental setup, the boundary layer approach is not effective in reducing the tracking error. Instead, we will present a scheme that adaptively adjusts the switching zone. In our experiments, we have found that this scheme consistently improves the tracking performance over our baseline controller. The controller currently in use in our testbed is a mass–matrix weighted proportional–derivative (PD)
controller which provides a baseline for comparison of the SMC results.

In design and implementation of any robot controller, it is critical to consider the following attributes of the overall system: discretization, joint compliance, nonlinear dynamic coupling, payload uncertainty, gravity load, sensor and actuator noise, and friction. For industrial arms, the limiting factors among the ones listed above are the last two: noise and friction; they are also the ones all too frequently ignored in theoretical and simulation studies. In terms of performance, beside arm stability, the figure of merit should include arm stiffness, arm vibration, disturbance rejection, trajectory tracking error, and steady state error.

In the literature on implementing SMC in robotics, there is little comparative study between various versions of SMC and other forms of manipulator control. Additionally, there is a need for the experimental evaluation of SMC under the severe implementation limitation typically found in industrial manipulators as described above.

In this work, we start with the standard SMC which starts to chatter even at 30% of the maximum torque level. This controller is then modified to include a boundary layer. It is argued and experimentally verified that this controller cannot outperform the baseline PD control. A combined PD and switching controller is then used and is shown to improve over the PD control alone and the standard SMC, if the level of the switching function is appropriately chosen. For this system, the presence of friction actually improves the controller response by reducing the conditions for chatter. Finally, an adaptive form of this controller is introduced and shown to be robust to friction and noise. This new controller produces improved performance over the baseline PD model-based control and provides unique insights into control of machines with friction through switching.

The robot model of the following form:

\[ M(q) \ddot{q} + D(q, \dot{q}) + g(q) = \tau \] (1)

where \( q, \dot{q}, \ddot{q} \) denote the joint displacement, joint velocity, and joint acceleration vectors, respectively; \( M \) is the inertia matrix, \( D \) contains the centrifugal and Coriolis forces, \( g \) is the gravity vector, and \( \tau \) is the joint input torque. After compensating for the gravity, the nominal sliding controller considers only the dynamics of the corresponding link and treats the coupling torques as disturbances. The controller is of the following form:

\[ \tau_i = \tilde{m}_{ii}(q) [\ddot{q}_i - c_i \Delta \dot{q}_i - \Delta \dot{q}_i \text{sign}(s_i)] + \dot{g}_i(q) \] (2)

where \( \tilde{m}_{ii} \) is the approximate i-th diagonal entry of the inertia matrix \( M \), \( \dot{g} \) is the approximate gravity compensation, \( s_i \triangleq \Delta \dot{q}_i + c_i \Delta q_i \), is the sliding variable and \( \text{sign} \) denotes the switching function with unit amplitude. If \( \dot{m} \) and \( \dot{g} \) are exact and the dynamics are decoupled (i.e., \( M \) is diagonal and \( D \) is zero), then the closed loop equation becomes

\[ \Delta \ddot{q}_i + c_i \Delta \dot{q}_i + \text{sign}(s_i) = 0 \] (3)

which implies that the origin of the error system is asymptotically stable; furthermore, this controller is robust with respect to modeling error and noise as has been amply demonstrated in the standard sliding control literature.

As discussed before, in this paper, we will consider various modification of this basic controller. To avoid chattering, a boundary layer is typically used instead of the infinite gain switching term in (5). With a boundary layer of width \( 2\phi \), the controller becomes

\[ \tau_i = \tilde{m}_{ii}(q) [\ddot{q}_i - c_i \Delta \dot{q}_i - K_i \text{sat}(\frac{s_i}{\phi})] + \dot{g}_i(q) \] (4)

where \( \text{sat} \) is the saturation function with unit amplitude and slope.

We have also augmented the boundary layer controller with a switching term, giving rise to the following controller:

\[ \tau_i = \tilde{m}_{ii}(q) [\ddot{q}_i - c_i \Delta \dot{q}_i - K_i \text{sat}(\frac{s_i}{\phi})] + \dot{g}_i(q) \] (5)

We then further replace the fixed gain in the switching term with an adaptive gain. This controller of the following form:

\[ \tau_i = \tilde{m}_{ii}(q) [\ddot{q}_i - c_i \Delta \dot{q}_i - K_i \text{sat}(\frac{s_i}{\phi})] - \dot{K}_i \text{sign}(s_i) + \dot{g}_i(q) \] (6)

\[ \dot{K}_i = -\alpha \dot{K}_i |s_i| \]

In this paper, we will report the experimental performance comparison of these controllers taking into account of transient and steady state tracking error, torque chattering, and arm vibration.

In Section 2, the experimental setup used in testing the various algorithms is described. In Section 3, we present the Sliding Mode Controller with Boundary Layer and show that it does not improve on the PD model-based controller. The PD plus switching controller with an analysis of the noise effect is presented in Section 4. Section 5 presents the PD plus an adaptive switching controller where the magnitude of the switching term is adapted through a first order adaptation law.
2 Experimental System

The experimental evaluations in this work were performed on a PUMA 560 under the real-time control system developed at CIRSSE [8]. The sampling rate for the experiments was 4.5ms or 222Hz sampling frequency. This sampling rate, and the test motions for the arm are similar to those used by Leahy [9]. In this study we also used a Minimum Jerk Trajectory but have slowed down the manipulator motions and reduced the range of motions. An evaluation of the manipulator use in the laboratory showed the need for better control at the somewhat slower robot motions than those used in [9] but at similar positions. The manipulator is moved through the positions that cause the peak gravity and inertial variations.

We examined four nominal arm positions and three sets of minimum jerk trajectories. The experimental results in this paper show joints 1 through 3 of the PUMA 560 for the extended nominal position of the manipulator using the minimum jerk trajectory with ±10 movement. All six joints of the PUMA were evaluated. While the smaller joints are much more difficult to tune due to their smaller mass and higher friction, the controller results presented in this paper held for all joints.

The desired motion lasted four seconds with an additional one second with constant position to assess the dynamic and static qualities of the closed loop control. The extended nominal joint position of the PUMA 560 was [0, 0, 90°, 0, 0, 0].

3 Sliding Mode with Boundary Layer

It is well known that the classic controller chatters in the presence of measurement noise, a first modification to this controller was the addition of a boundary layer. When the position and velocity errors are small, the controller will operate inside the boundary layer. Thus, without large disturbances, a correctly operating controller will never leave the boundary layer. Figure 1 shows the operation of the Sliding Mode Controller with a Boundary layer. The plot in the lower left corner shows the phase plane along with the boundary. This controller, without disturbances, operates continuously in the boundary layer.

In (4), the \( \text{sat} \) function becomes linear in \( s \) when in the boundary layer. It is straightforward to show that the resulting controller is exactly the PD form with model information. Figure 5 shows the \( \text{sat} \) function for the sliding mode with a boundary on the left and the PD Switching controller on the right. The PD Switching controller will be presented in a following section. It is important to note that the controller operates with \( s \) within the 2\( \delta \) region. The critical element in the operation of this controller is the slope of the linear portion of \( \text{sat} \) function in the boundary region. The higher the slope, the higher the equivalent PD gains, and the better overall tracking and steady-state performance.

However, it is well known that in the digital implementation of PD control systems, there is a set of PD gains beyond which the system is unstable. This limit sets the upper bounds for the slope in the Sliding Mode Controller and the gains in the PD model-based controller. Because the Sliding Mode controller in the boundary region is identical to the model-based PD controller, both controllers have the same effective limit. Additionally, the saturation point, \( K_{sat} \), for the sliding mode controller is at best lower or equal to the saturation point of the physical motors. This becomes an artificially induced saturation on a PD type controller. The sliding mode with a boundary will reach the saturation point sooner than a PD controller. Thus sliding mode with a boundary cannot out perform the model-based PD controller.

Figures 1–4 show the tracking error and the joint torques for joints 1–3 of the PUMA 560 manipulator under this controller. While this controller has no chattering problem, the response is no better that the baseline PD controller used in the laboratory. The baseline analysis for joint 1 is shown in Figure 2.

![Figure 1: Analysis of Sliding Mode with Boundary for Joint 1.](image)

![Figure 2: Analysis of PD Model Based Control for Joint 1.](image)
4 Sliding Mode/PD with Switching

The appeal of the switching type sliding controller lies in its theoretically infinite gain which can reduce the tracking and steady state error to zero. However, the gain $k_{sign}$ must be limited to a fraction of the full range in order to avoid excessive vibration of the arm.

The addition of the boundary layer removes the torque chattering and allows the full utilization of the joint motor. However, as seen in the previous section, this strategy does not improve the closed loop performance over the PD controller due to the gain limitation within the boundary layer.

In this section, we will combine the two controllers (cf. (5)) with the goal to appropriately select the parameters to reap the benefit of both. The controller structure consists of a PD summed with a switching component. The basic idea is to choose the switching gain so that $s_i$ is reduced from the PD case but not to zero. The most desirable level of $s_i$ should be just beyond the noise level so torque switching cannot occur at all.

Figure 5 illustrates the impact of noise on the joint torque in the PD plus switching controller versus the boundary layer controller. Assume that the gain with in the boundary layer is tuned optimally based on the consideration presented in the previous section, i.e., the slope shown in the linear region is at its maximum. Consider the situation when the low frequency component of $s_i$ stays in the vicinity of $s_i = 0$ but is still far enough far from the sliding line so that $s_i$ does not cause torque switching. Then the PD plus sliding controller contains the same amount of noise as the boundary layer controller since the gain within this region is the same. However, PD plus sliding controller produces a higher average torque than the boundary layer controller because of the torque offset in the switching term. Thus with this enhancement, we have obtained the desired result of increasing the torque without increasing the amount of noise in the system.

It is important to select $k_{sign}$, so that $s_i$ operates primarily in the linear region (i.e., away from zero by at least the amount of noise). Otherwise, as illustrated in Fig. 8, the noise actually has a larger adverse influence in the PD plus switching case. Adjusting the gain $k_{sign}$ is currently done heuristically. A gradual increase in $k_{sign}$ from zero to about half the joint stiction value showed a sizable impact on the time necessary to break free. Here we have used only a crude estimate of the stiction torque, which was found by plotting torque vs. velocity in the boundary layer case and assessing the torque value at which the joint broke free.

The experimental results of tracking the same minimum jerk trajectory as in the previous section using the PD plus switching controller are shown in Fig. 6 and Fig. 7. The first plot shows the joint position tracking error of the first three joints and the second plot shows the corresponding torques. The performance in terms of both tracking error and steady state errors is much better than the PD case alone. In the case of joint 3, the steady state error is almost zero. However, some torque chatter does occur in joints 2 and 3, especially in the steady state region for joint 3. This clearly illustrates the trade-off between the level of position error and the torque level which is a function of $k_{sign}$. It also shows the difficulty of choosing...
$k_{s_{ign}}$, so a reasonable compromise between error and noise induced chattering can be reached. Indeed, the acceptable level of $k_{s_{ign}}$ is trajectory and configuration dependent. Furthermore, a payload changes or a persistent load disturbances can induce chatter by forcing $s_i$ too close to zero. Therefore, even though this method can show improvement over the baseline controller with a carefully tuned gain, it is not practical due to the lack of robustness. In the next section, we will show a simple modification that provides a partial resolution of this issue.

![Graph](image)

Figure 6: Position Error Data of nonadaptive PDS for Joints 1-3, J1=solid, J2=dashed, J3=dashdot.

![Graph](image)

Figure 7: Torque Data of nonadaptive PDS for Joints 1-3, J1=solid, J2=dashed, J3=dashdot.

5 Adaptive Sliding Mode/PD

The PD with switching controller shown in the previous section works satisfactorily if the switching gain $k_{s_{ign}}$ is chosen so that $s_i$ is kept away from zero with a distance at least above the noise level. However, this value is not always known a priori. Furthermore, the appropriate value is configuration dependent and can change due to a variation of the external load. If the chosen gain is too high, torque choking once again occurs. Fig. 8 illustrates the effect of noise when $s_i$ is within the noise region. Now the SMC with boundary layer actually produces a smaller torque swing. A good example for this case is when a load is attached to the gripper and the arm itself is moving downward. If the gravity compensation is only for the unloaded arm, the additional gravity effect of the mostly unknown load is enough to drive $s_i$ close to zero with torque chattering as result. For Joint 3 in the above example, this situation occurs even while holding the joint at a constant angle.

In this section, we present an intuitive solution to this problem. By again comparing Figures 5 and 8, "adaptation" of $k_{s_{ign}}$ appears to be a reasonable compromise between the two extremes. Whenever the sliding error $s_i$ is small the gain $k_{s_{ign}}$ should be also small. For a large $s_i$, we should boost $k_{s_{ign}}$ to guard against further increase of $s_i$. This strategy can be accomplished by adjusting the switching gain through a first order filtered version of $s_i$. The analytic form of this controller is given in (6).

![Diagram](image)

Figure 8: Noise Diagram II for $\tau_{s_{ign}}$ of SMC-BLCT and $\tau_{s_{ign}}$ of PDS.

As in any control law, the closed loop performance depends on a large extend on the selection of gains. So far, we have used a set of heuristic strategies which appear to consistently produce good results. Basically there are three parameters to select:
1. Initial condition of the filter.
2. Filter constant $\alpha$.
3. Filter gain $K_f$.

These parameters are chosen based on the following rules:
1. The initial values for $k_{s_{ign}}$ are chosen to be the gains of the non-adaptive case.
2. Choose the filter cut-off frequency to be twice as high as the cut-off frequency of the joint velocity filter. This is an empirical choice attempting to strike a balance between performance (response of gain to the change in $s_i$ and noise (effect of noise in $s_i$ on the gain). The cut-off frequencies for the PD case are:
   - Joint 1-3: 54 rad/sec
   - Joint 4-6: 105 rad/sec
   - The cut-off frequencies selected for the adaptive gain filter are:
     - Joint 1-3: 108 rad/sec
     - Joint 4-6: 210 rad/sec
     - 3. If $s_i$ is a constant and is at the peak of the noise level, the steady state gain $K_{st}$ is chosen to be the same as the non-adaptive case to avoid chattering.
These rules allow the resolution of the three parameters. Repeated experimentation has shown the apparent effectiveness of this heuristic strategy.

The experimental results of tracking the same minimum jerk trajectory are shown in Fig. 9 and Fig. 10. The first plot shows the joint position tracking error of the first three joints and the second plot shows the corresponding torques. Compared to the PD and fixed switching zone case, it is evident that all torque switching have been removed. This is especially evident in joint 3. As a consequence, the tracking and steady state errors are larger. However, compared to the PD case, these errors are reduced by at least half in all joints.

![Figure 9: Position Error of Adaptive Sliding Mode for Joints 1-3, J1=solid, J2=dashed, J3=dashdot.](image1)

![Figure 10: Torque Data of Adaptive Sliding Mode for Joints 1-3, <Traj 1>, <Extend>, J1=solid, J2=dashed, J3=dashdot.](image2)

6 Conclusion

This paper presents an experimental study that compares the performance of a baseline PD controller with several versions of sliding mode controller, modified to avoid the common torque chattering problem. It is found that the standard boundary layer modification is essentially the same as the PD controller and does not offer any inherent advantage. A PD plus switching controller offers improved transient and steady state position error and avoids chattering with an appropriately chosen size for the switching term. This switching level can be further adapted by the level of tracking error smoothed by a simple first order filter. This controller offers the best compromise between performance objective, torque chattering, and robustness with respect to the external load. Future work will include the addition of friction compensation, and consideration of sliding control in the context of position accommodation type of force control.

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References


