Preisach modeling of piezoceramic and shape memory alloy hysteresis

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Abstract. Smart materials such as piezoceramics, magnetostrictive materials, and shape memory alloys exhibit hysteresis, and the larger the input signal the larger the effect. Hysteresis can lead to unwanted harmonics, inaccuracy in open loop control, and instability in closed loop control. The Preisach independent domain hysteresis model has been shown to capture the major features of hysteresis arising in ferromagnetic materials. Noting the similarity between the microscopic domain kinematics that generate static hysteresis effects in ferromagnetics, piezoceramics, and shape memory alloys (SMAs), we apply the Preisach model for the hysteresis in piezoceramic and shape memory alloy materials. This paper reviews the basic properties of the Preisach model, discusses control-theoretic issues such as identification, simulation, and inversion, and presents experimental results for piezoceramic sheet actuators bonded to a flexible aluminum beam, and a Nitinol SMA wire muscle that applies a bending force to the end of a beam.

1. Introduction

Actuators and sensors based on materials such as piezoceramics or shape memory alloys (SMAs) exhibit significant hysteresis. Hysteresis is a form of nonlinearity that contains memory, thus there may be multiple possible outputs for a given input. Unmodeled hysteresis can lead to inaccuracy in open loop control, and generate amplitude-dependent phase shifts and harmonic distortion that reduce the effectiveness of feedback control.

The strongest hysteretic effects are observed when the input signal undergoes full scale excursions. At the other extreme, very small inputs result in an approximately affine relationship between the input and output; the system well approximates a constant offset plus a constant gain. Thus harmonic distortion and input amplitude-dependent gain and phase shifts approach zero. The values of the offset and gain are, however, dependent upon the current state of the hysteresis, which depends upon the previous inputs.

There is a vast literature on hysteresis modeling, especially for ferromagnetic materials, but a relatively small portion of this is related to piezoceramic or SMA materials. Furthermore, the models are usually developed for physical based simulation rather than for stability analysis and control design. The physics of piezoceramic and SMA hysteresis bears a strong resemblance to that of ferromagnetic materials—the underlying action is the restriction of coherent domain dynamics by static frictional forces, see, e.g. [1] for ferromagnetics, [2] for ferroelectrics, and [3] for SMA. Therefore, tools developed for ferromagnetic hysteresis may be applicable to piezoceramic and SMA as well.

The most popular hysteresis model for ferromagnetic materials is the Preisach hysteresis independent domain model [4]. This model has many well defined properties (see [5] for a comprehensive exposition) that make it suitable for control applications.

The Preisach model is completely characterized by two properties, the minor-loop congruence property and the wiping-out property. A minor loop is the closed loop in the input–output graph when the input cycles between two extrema. Two minor loops are said to be comparable if they are generated by inputs with the same extrema. The minor-loop congruence property states that all comparable minor loops are identical in shapes. The wiping-out property specifies the values of the past input trajectory that affect the current output; in particular, a large dominant extremum 'wipes out' the effect of past smaller dominant extrema. These two properties can be checked experimentally, to determine if a physical hysteretic process may be matched by a Preisach model.

There has been limited success in applying the Preisach model to piezoceramics and SMAs. In [6], a piezoceramic bimorph was tested for Preisach hysteresis. Minor-loop congruency was demonstrated for symmetrical minor loops. In [7], the standard Preisach model was applied to a single large crystal of Cu–Zn–Al (a cheaper material than Nitinol, but one which exhibits more pronounced hysteretic effects). This work demonstrated the approximate congruency of
minor loops and the dependency of the current output on previous dominant input extrema. The input-output variables were chosen to be stress and strain, respectively, and all experiments were performed at an approximately constant temperature.

In this paper, we review some key properties of the Preisach model including simulation and identification, and derive an efficient inversion algorithm. These tools are then applied to piezoceramic and SMA actuators in our laboratory.

The rest of the paper is organized as follows. Section 2 reviews the physical mechanism behind hysteresis found in ferromagnetic, ferroelectric (including piezoelectric), and shape memory materials. Section 3 presents the Preisach model and summarizes its key properties. Section 4 presents experimental results related to applying the Preisach model to piezoceramics and SMAs.

2. Background

2.1. Ferromagnetism and the Preisach model of hysteresis

Hysteric effects have been observed in many physical systems, e.g. ferromagnets, ferroelectrics (and the subset of piezoceramics), plastic materials, gas adsorption systems, optical devices, chemical systems, SMAs, and others. Ferromagnetic hysteretic materials in particular have long been studied. One of the most successful mathematical models is the Preisach independent domain model first proposed in 1935 by Preisach [4] as a physical representation, but it was subsequently realized that it is more phenomenological in nature [5].

The Preisach model is completely characterized by two properties, the wiping-out property (only the current input and certain previous input extrema affect the current output) and the congruent minor-loop property. These will be explained later in greater detail. The Preisach model has several appealing features including its ability to model complex hysteresis types, a well-defined identification algorithm, and a convenient numerical simulation form.

In the 1970s and 1980s, the mathematical properties of the Preisach model were examined and developed by the Russian mathematician Krasnosel’ski[8]. Krasnosel’ski separated this model from its physical meaning and represented it in a purely mathematical form which is similar to a spectral decomposition of operators.

Recently, others have studied the Preisach model and proposed several extensions to overcome its limitations when applied to ferromagnetic materials; these include relaxation of the congruent minor loop requirement [9–11], dependence on output rate of change [12], experimental validation of the Preisach model [13], and application to integral ferromagnetic problems [14].

2.2. Ferromagnetism and ferroelectricity

Ferromagnetic solids are magnetic materials with a large positive magnetic susceptibility and thus they readily adopt a parallel magnetization in the presence of an applied magnetic field. Within these materials, individual atoms behave as magnetic dipoles, and a large number of these, typically $10^{12}$ to $10^{15}$, align parallel in cluster groups called domains [1]. The magnetic orientation of each domain can differ from its neighbors (they are separated by imaginary ‘domain walls’) and hence a zero, or an approximately zero, net magnetization may result. Application of a magnetic field causes domains aligned with the field to grow (by adopting newly aligned neighboring atoms, a predominantly reversible effect), and the remaining ones to correspondingly shrink; the domain walls are said to move in response to the applied field. However, defects such as dislocations interact with the magnetic dipoles and obstruct the domain wall motion by pinning. When this occurs, unrecoverable energy is lost from the applied field in breaking them free, no matter how slow the process, and this is the primary source of hysteresis.

Ferroceramics are polycrystalline materials that contain many individually oriented ferroelectric grains (crystals), and, analogous to ferromagnetic materials, within each grain there are clusters of aligned spontaneously electric dipoles called domains, again separated by imaginary domain walls. Initially such ceramics are not piezoelectric even if the individual grains are; application of a small electric field will cause some electric dipoles to grow, and others to shrink, so that there is no net change in strain [15, p 156]). However, upon application of a sufficiently large electric field, some of the electric dipoles will switch directions (and, as a result, the domain walls will move), and there is now a net piezoelectric effect; this process is called poling.

Again analogous to ferromagnetic materials, domain walls may be pinned by material defects, and this is believed to be the primary source of hysteresis. Supporting this theory, a mathematical theory has been developed [2] based on a model of random interactions between lattice defects and domain walls in ferroelectric materials. This theory has successfully estimated several characteristic parameters of a hysteresis loop such as its initial dielectric constant and the coercive field value.

The success in applying the Preisach model to ferromagnetic hysteresis, and the similarity between the primary hysteresis mechanisms in both ferromagnetic and ferroelectric materials suggests that the Preisach model may also be a suitable one for representing piezoceramic hysteresis.

The first application of the classical Preisach model to a piezoceramic material appeared in [6]. The identification and numerical simulation procedures detailed in [5] was used to model the hysteresis exhibited by a piezoceramic bimorph. The minor-loop patterns and enclosed areas (representative of cyclic energy losses) from simulation and experimental data matched well for symmetrical minor loops. Recent extension using the extended Preisach models can be found in [16].

2.3. Other ferromagnetic and ferroelectric hysteresis models

The Duhem hysteresis model [17, 18], proposed at the turn of the century, has been applied to some ferromagnetic
materials [19]. However, it has been shown [20] that it is a poor match for a Mn–Al–Ge magnetic film, while an extended Preisach model performs very well. In general, the Duhem model is less complex than the Preisach model, and it does not allow the crossing of minor loops which can arise in ferroelectric materials. The Duhem model does model elastoplasticity quite well [17].

The Stoner–Wohlfarth (S–W) model [21] is considered a physical model for ferromagnetics since it represents a collection of single-domain uniaxial magnetic particles, and it is a popular vector hysteresis model in magnetics. However, it has several serious deficiencies. Since the basic building block is a symmetric hysteresis, the S–W model cannot represent non-symmetric minor loops. Expanding the set of elementary operators to include asymmetrical ones can correct for this, but the result is to complicate the original S–W model which is already computationally inefficient. The parameter identification in S–W model is also not well developed.

2.4. SMA hysteresis models

The shape memory effect refers primarily to the manner in which certain materials can be deformed in a low-temperature martensitic crystalline state yet always return to the same unique shape in a high-temperature austenitic (parent) phase [22]. A popular such material, and the one used in our experiments, is called Nitinol and it is an approximately 50–50 mixture of nickel and titanium [23]. At higher temperatures, all the material is in the austenite phase. As the temperature is lowered, areas of martensite appear and grow until at sufficiently low temperatures all the material is in the martensite phase. Stress and strain can also affect such a transformation, and can control the exact nature of the martensite phase through the process of twinning. The temperature input is analogous to the electric field in ferroelectrics, and the magnetic field in ferromagnetics. The two main sources of hysteresis in SMA are static frictional effects associated with twinning and detwinnning processes (which can be considered as a form of domain wall pinning by ‘defects’ with the twinning interfaces as domain walls and crystal cells along the twinning interfaces as defects), and those due to lattice defects [3, 22]. The similarities to the ferromagnetic and ferroelectric hysteresis mechanisms suggest that a form of Preisach model may be appropriate. In [24], a form of a Preisach model is applied to SMA by using a four parameter (and relatively complex) ‘Landau–Devonshire’ model element to represent individual SMA crystals, rather than using the standard two-parameter relay elements. The model is compared to experimental data (for an unspecified polycrystalline material) and mixed results are obtained; the stress–strain curves were of a similar nature, but predicted parameters such as the Curie temperature differed significantly. The control parameter was stress and the observed parameter was strain; all the experiments were performed at approximately constant temperatures.

In [7], the standard Preisach model is applied to a single large crystal of Cu–Zn–Al (a cheaper material than Nitinol, but one which exhibits more pronounced hysteretic effects) and obtained a good match demonstrating the approximate congruency of minor loops and the dependency of the current output on previous dominant input extrema. Again the control parameter was stress, the observed parameter was strain, and all the experiments were performed at an approximately constant temperature.

3. Preisach model

The basic Preisach model consists of a weighted combination of elementary relay elements (see figure 1). The model can be mathematically described as

\[ f(t) = \sum_{(\alpha, \beta) \in S} \mu(\alpha, \beta) \gamma_{\alpha\beta}(u(t)) \, d\beta \, d\alpha \]

where \( \mu(\alpha, \beta) = \nu(\alpha, \beta) + \kappa(\beta) \delta(\alpha - \beta) \) and \( S = \{(\alpha, \beta) : u_{\text{min}} \leq \beta \leq \alpha, u_{\text{min}} \leq \alpha \leq u_{\text{max}} \} \). The weighting function \( \nu(\alpha, \beta) \) captures the non-hysteretic, memoryless input–output relationship. The weighting function \( \nu(\alpha, \beta) \) contains the hysteretic effect.

Initially the Preisach model was proposed as a physical model for ferromagnetic hysteresis effects, each elemental relay operator representing one magnetic dipole. However, there are two major shortcomings:

1. Magnetic dipoles exhibit only a symmetrical response with respect to the magnetic field input, and therefore \( \alpha = -\beta \) for all dipoles.
2. The state of an individual dipole is determined by the applied magnetic field and the combined effect of all the other dipoles, but the basic Preisach model does not directly model such an interaction.

Therefore, the Preisach model is only a phenomenological model, although it is physically motivated.

The output of the Preisach model is the integral of \( \mu(\alpha, \beta) \) weighted by +1 or −1 (the relay outputs) depending on the past input history. These binary weightings are best visualized directly in the \( \alpha–\beta \) plane as shown in figure 2. It is very useful to correlate the behavior of the Preisach model with the changes in the relay outputs. As \( u(t) \) increases, a vertical line intersecting the \( \alpha \) axis at \( u(t) \) sweeps in the positive \( \alpha \) direction in \( \alpha \), changing the relay outputs to the left of the line all to +1. As \( u(t) \) decreases, a horizontal line intersecting the \( \beta \) axis
at \( u(t) \) sweeps in the negative \( \beta \) direction in \( S \), changing the relay outputs above the line all to \(-1\).

As an example consider a sample input \( u(t) \) and the corresponding relay outputs shown in figure 2. The input \( u(t) \) is initially lowered to its minimum, \( u_{\text{min}} \), setting all relays to \(-1\). As the input increases to \( u_1 \), a vertical line sweeps along the positive \( \alpha \) axis changing relay outputs to \( +1 \). The input is then lowered to \( u_2 \); a horizontal line sweeps down the \( \beta \) axis changing some relay outputs back to \(-1\). When the input is raised again to \( u_3 \), more relays are changed to \(+1\). The output at each time instant is simply the integral of \( \mu(\alpha, \beta) \) weighted by the corresponding relay outputs.

It follows from such an analysis that the Preisach plane \( S \) at any instant may be divided into two parts, \( S_1 \) where the relay outputs are \( 1 \) (dark grey), and \( S_2 \) where the relay outputs are \(-1 \) (light grey). Furthermore, the two sections are separated by a descending staircase line. The corners of this staircase are determined by the previous reversal points of the input.

It has been shown [5] that the necessary and sufficient condition for a hysteresis nonlinearity to be represented by the basic Preisach model is that the wiping-out property and minor-loop congruence property hold. These two properties are explained below.

**The wiping-out property.** The wiping-out property refers to the constraint that the output be affected only by the current input and the alternating series of previous dominant input extrema, the effect of all other inputs being wiped out. A dominant maximum is one that is greater than any subsequent \( u(t) \), while a dominant minimum is one that is less than any subsequent \( u(t) \).

This effect can be most easily seen from the relay outputs in \( S_1 \) and \( S_2 \) Preisach plane. As illustrated in figure 2, the relay outputs only depend on the current input, \( u(t) \), and past maxima and minima which determine the corners of the staircase dividing line. Furthermore, if \( u_3 > u_1 \) in the rightmost diagram then the effect of \( u_1 \) on the shapes of \( S_1 \) and \( S_2 \) (and thus on hysteresis output) is effectively erased. When \( u_2 \) grows larger than \( u_1 \) it is said to wipe out the memory of \( u_2 \). An analogous situation is seen when the input falls below a previous dominant minimum.

**The congruent minor-loop property.** The congruent minor-loop property requires that all equivalent minor hysteresis loops be congruent (see figure 3). Two minor loops are said to be equivalent if they are generated by an input \( u(t) \) varying monotonically between the same two extrema. Congruency between two minor loops means that one will exactly overlap the other if shifted by an appropriate vertical translation. The congruency property implies that the shape of a minor loop depends only on the two extremum values of the input path used to generate the loop.

This property can also be seen from the relay output diagram in the \( \alpha-\beta \) plane. As an input goes monotonically from \( u_1 \) to \( u_2 \) and then back to \( u_1 \), the change of the relay output is the same irrespective of the initial conditions. The only effect of the initial condition is in producing a bias in the output.

### 3.1. Identification

In [5], a procedure for identifying the weighting function \( \mu(\alpha, \beta) \) has been proposed. The basic idea is to identify a two-parameter function \( f_{\alpha, \beta} \) defined as the Preisach output with the relay state as shown in figure 4:

\[
f_{\alpha, \beta} = \int \int \mu(\alpha, \beta) \, d\beta \, d\alpha - \int \int \mu(\alpha, \beta) \, d\beta \, d\alpha.
\]

It is straightforward to show [5] that

\[
\nu(\alpha, \beta) = \frac{1}{2} \frac{\partial^2 f_{\alpha, \beta}}{\partial \alpha \partial \beta}, \quad \kappa(\alpha) = \frac{1}{2} \frac{\partial f_{\alpha, \beta}}{\partial \beta} \bigg|_{\beta^+}.
\]
that the Preisach output can be expressed as:
\[
\ddot{u}(t) \leq 0 \Rightarrow f(t) = f_{\min} + \sum_{l=1}^{n(t)-1} [f_{M_l,m_{l-1}} - f_{M_l,m_{l+1}}]
\]

where \( f_{\min} \) is the output corresponding to \( u = u_{\min} \) (all relays set to \(-1\)). This formula can be used for efficient simulation as the output is calculated directly from measured data and neither integration nor differentiation is required.

For discrete time simulation, \( f(t) \) is replaced by \( f(q \Delta T) = f_q \) and \( u(t) \) is replaced by \( u(q \Delta T) = u_q \) where \( t_0 = 0 \Rightarrow q = 0 \) and \( \Delta T = 1/F; F \) is the sampling frequency. Then at each sampling instance the new dominant extrema sets \( M \) and \( m \) are updated and a new \( f_q \) evaluated.

Two important implementation issues are how large the sets \( M \) and \( m \) grow and how the updates to be carried out at each sampling time. If one considers an exponentially decaying sinusoid input then after each 'cycle' a new dominant maxima, and a new dominant minima are added so that \( M \) and \( m \) grow without bound. An approximate solution to this problem would be to quantize the input data so that only one of, say, \( 2^p \) levels are allowed, and then the maximum length of \( M \cup m \) would be limited to \( 2^p \). The output errors introduced by doing so would be bounded since individual dominant extremum errors are bounded, there are a limited possible number of dominant extrema, and an analytical first-order curve surface is assumed. An application where such quantization occurs naturally is when reading a hysteretic sensor signal via an analog-to-digital converter.

Updating \( M \) and \( m \) each sampling cycle can be a relatively simple process if one successively compares any new dominant extrema with previous dominant extrema and then deletes none, one or more previous dominant extrema due to wiping out, and then adds the new dominant extrema. Since a data quantization resolution of \( 2^{12} \) was used so that there were theoretically 4096 possible dominant extrema at any one time, of which 4095 could be all maxima, or
all minima (only $M$ or $m$ need be examined at any time interval depending on whether $u_{1-n}$ was a peak or a valley) and so other search algorithms may be preferred in order to guarantee a sufficiently fast program.

3.3. Numerical Preisach model inversion

The inverse Preisach model may be readily derived from (4) and (5) if the first-order curve data surfaces, $f_{\alpha, \beta}$, are strictly monotonic with respect to their two parameters $\alpha$ and $\beta$. This has been observed to be the case for the piezoceramic and SMA actuators tested.

Define $F(\alpha, \beta) = \frac{1}{2} (f_{\alpha, \beta} - f_{\alpha, \beta})$. This function corresponds to the relay diagram shown in Figure 6. Define the inverse of $F(\alpha, \beta)$ with $\beta$ or $\alpha$ fixed $G_\alpha(.,.)$ and $G_\beta(.,.)$, respectively: given $z = F(\alpha, \beta)$, $\alpha$ can be written as a function of $\beta$, $\alpha = G_\alpha(z, \beta)$, and $\beta$ can be written as a function of $\alpha$, $\beta = G_\beta(\alpha, z)$. Applying this to (4) and (5), we obtain $u(t)$ needed to produce a desired output $f(t)$:

\[\dot{u}(t) \geq 0 \Rightarrow u(t) = G_\beta \left( M_n, \left[ -\frac{f(t) + f_{\min}}{2} + P_n + F(M_m, m_{n-1}) \right] \right) \quad (6)\]

\[\dot{u}(t) \geq 0 \Rightarrow u(t) = G_\alpha \left( \left[ \frac{f(t) - f_{\min}}{2} - P_n \right], m_{n-1} \right) \quad (7)\]

where $P_n = \sum_{k=1}^{t} \left[ F(M_k, m_{k-1}) - F(M_k, m_k) \right]$.

For discrete time implementation, $u(t)$ and $f(t)$ are replaced by $u_q$, $f_q$ at each sampling time. The output dominant extrema sets $M_f$ and $m_f$ ($M_f = f(M_i)$ and $m_f = f(m_i)$ are the outputs corresponding to the past dominant maxima and minima, respectively) are updated and a new $u_q$ can then be calculated. As before, amplitude quantization of the input (i.e. of $f_q$) would guarantee a finite limit to the number of elements in $M_f$ and $m_f$.

4. Experimental results

4.1. Hardware overview

Figure 7 is an outline of the physical single flexible link tested. The central component is a flexible beam which is chosen to provide readily measurable flexibility in the horizontal plane that is independent of gravity (ideally).

This beam is driven by three types of actuators: a DC motor at the hub, two sets of piezoceramic patches approximately colocated with the outer two strain gages, and a Nitinol SMA wire muscle attached between the hub and the free end of the beam. This beam is made from aluminum and has the dimensions $109.8 \times 10.3 \times 0.16$ cm.

The real-time computer is a networked set of four Inmos Transputer micro-processors which are connected to a PC host for software development and user interface. These Transputers also connect to a VME bus on which resides the digital to analog (D/A) and analog to digital (A/D) boards providing the actuator and sensor interfaces.

4.2. Piezoceramic actuators

The piezoceramic patches (or sheets) are driven by high-voltage–low-current amplifiers (supplied by Piezo Systems Inc) which apply a control voltage across the thickness of a patch causing a parallel extension (if the voltage is the same sign as the poling voltage) and simultaneously a perpendicular contraction (in a two-dimensional plane parallel to the plane of the beam), and vice versa for a reversed voltage.

Each piezoceramic set consists of four patches and two are mounted on each side of the beam above and below each of the outer two strain gages. The piezoceramic material is Lead Zirconate Titinate (PZT for short) made by American Piezo Ceramics, type 8-50, (equivalently Navy type II, or Piezo Electric Products type G-1195), the dimensions are $2'' \times 1.2'' \times 0.01''$, and the two large surfaces are silvered.

These patches are glued to the beam using a preformed electrically conductive epoxy glue that is supplied in thin sheets (0.002' thick) and so a very uniform bond can be achieved. The curing temperature cycle used is $120^{\circ}$C for 30 min approximately; this was achieved by using a heat gun with the surrounding air temperature at $21^{\circ}$C approximately.

4.3. Shape memory alloy wire muscle

The SMA wires consist of 0.008' thick Nitinol wires, approximately 1.09 m long. These are Flexinol wires made by Dynalloy with a characteristic transition temperature given as $90^{\circ}$C and a maximum pull force of 590 g approximately. The ambient room temperature for the SMA experiments was $20^{\circ}$C approximately. A complication of this configuration is that the wire stress is a function of the wire strain (and thus the beam strain) and so independent control of these two variables is not possible once the wire is connected unless an external force is applied, e.g. by use of weights pulley wheel, or by use of a second SMA wire muscle on the other side of the beam (fixtures have been installed for this purpose).

4.4. Piezoceramic tests

For the piezoceramic tests the controlled parameter was transverse electric field (i.e. the voltage applied across the thin dimension of the piezoceramic sheets divided by the nominal thickness), and the observed parameter was the approximately colocated strain measured with a standard.
film strain gage. In each test, a smooth continuous low rate of change command trajectories were used in order to avoid exciting flexible beam modes to any significant extent.

4.4.1. Minor loop tests. Figure 8 (left) shows the applied electric field trajectory for the first of two minor-loop experiments, and it is composed of scaled and translated sinusoidal segments: the zero slope points are marked from 1 to 12 and they correspond to maxima, minima, or points of inflection. The first minor loop is traced from points 4 to 6 and the second minor loop (comparable to the first) is traced from points 9 to 11. In between these minor loops a large maxima occurs at point 7. The induced beam strain is shown in figure 8 (right) and an effect of the intervening dominant maxima can be seen in that the second minor-loop response is higher than the first. This is more clearly seen in the corresponding input-output (I/O) graph in figure 9 (left). Here the bounds of the comparable minor-loop input are marked by two vertical segmented lines and in figure 10 (left), these minor loop input–output graphs are shown in isolation and magnified. The characteristic analog-to-digital (A/D) converter staircase effect is seen (a 12 bit A/D converter was used) along with some noise or error due perhaps to small oscillations of the beam; however, a high degree of underlying congruency may be seen. The same data is also shown in figure 10 (right), after a narrow (10 point) raised cosine window is convolved with the input data, and with the output data, in order to smooth out the error effects and bring out the underlying loop shapes. The minor-loop congruency is now more evident than before; however, the upper minor loop is slightly ‘wider’ than the lower one. If one uses an analogy with the ellipse the ‘minor’ axis is seen to be slightly longer for the upper minor loop (note, however, that these minor loops contain slope discontinuities at their input extremities and are thus not ellipses).

In figure 9 (right) the input output graph generated by applying the inverse of the previous input electric field trajectory to the piezoceramic actuator is shown, and in this case two minor loops are traced with an intervening dominant minima. Again a high degree of congruency is seen and upon closer examination the lower minor loop proves to be slightly wider than the upper one.

4.4.2. Decaying sinusoid tests. Figure 11 (left) shows the applied electric field trajectory for the decaying sinusoid experiment designed to test for wiping out behavior. The input is brought through the largest minor loop available (limited by the amplifier) in order to wipe out previous dominant extrema and then a linearly (as opposed to exponentially) decaying sinusoid is applied in all producing an alternating sequence of ten maxima and minima before the signal is repeated.

The induced beam strain is shown in figure 11 (right) and the horizontal segmented lines have been added to demonstrate the close match between the first set of maxima and the second set indicating that the effect of the former has been wiped out by the first large maxima of the latter. More convincingly, figure 12 (right) shows the corresponding input–output graph and a very good match between the first half and the second half is seen; indeed it is difficult to tell them apart, again indicating that the effects of the first set of dominant extrema being wiped out.

Figure 11 (right) also has five vertical segmented lines added at the time locations corresponding to the input trajectory reaching each of the previous dominant maxima as it rises in the second half. If the wiping-out property holds then we can expect the intersection of the horizontal and vertical lines to match the rising output trajectory. Figure 12 (left) is a closer view of this rising output and a fairly good match is seen, although the rising output does ‘lag behind’ (i.e. lie below) these intersections slightly. Again the analog-to-digital (A/D) converter staircase effect is seen.
4.4.3. Identification. In initial experiments the piezoceramic voltage is controlled and the approximately colocated strain gage is used as a sensor, and in the quasi-steady-state case the linear equation is

$$V_s = K(E_p d_{31}) V$$  (8)

where $K$ is a constant given in [25]. The hysteretic effects are contained in the piezoceramic $E_p d_{31}$ term which is a

linear approximation. A set of eleven evenly spaced first-order descending curves are used to identify this hysteresis and are shown in figure 13 (left). A third-order, two-dimensional surface is then fitted to the corresponding set of monotonically decreasing responses using least-squares estimation as shown in figure 13 (right). The next step is to derive the weighting function for the Preisach model using
which results in a new two-dimensional surface (first order), $v(\alpha, \beta)$ as well as a second-order, one-dimensional curve, $\kappa(\alpha)$ along the ($\alpha = \beta$) diagonal line that represents the initial slopes of the first-order descending curves, see figure 14 left and right, respectively. Once the function $f_{a, \beta}$ is identified in an analytic form, it can be used for efficient simulation as described by equations (4) and (5). In figure 15 we compare the simulated and measured outputs (left) and input-output graphs (right), respectively, for a decaying sinusoid input and a very good match is seen (note that the simulated I/O graph is shown twice, once by a dashed curve, and the second time by a full curve shifted down for clarity). Similar results are obtained when the inputs for the first-order descending curve tests are used.

4.4.4. Direct compensation with inversion. Once the first-order descending curve surface is identified the inverse Preisach model may be numerically implemented using (6) and (7). Now the dominant extrema are detected from the desired output trajectory rather than the input trajectory. This inverse model was implemented in Matlab code (for simulation) and in Transputer OCCAM code for real-time tests. Figure 16 (left) shows a decaying sinusoid electric field applied to the piezoceramic actuator, and a scaled version applied as a desired strain to the inverse Preisach hysteresis algorithm in a real-time test; the output of this algorithm was then sent to the piezoceramic actuator. Figure 16 (right) then shows the resulting input–output graphs for the two cases and the lower one corresponding to the inverse hysteresis compensation test (shifted down for the sake of clarity) is seen to be much more linear than the uncompensated version.

4.4.5. Stress-dependent results. To evaluate the effect of large stress on the hysteresis characteristics, we apply a large static deflection force to the tip of the beam. The externally induced stress on the piezoceramic patch is at least an order of magnitude larger than that generated by the piezoceramic actuator alone in the static case. Such large stresses may be generated by the same piezoceramic actuator when excited close to a beam resonant frequency, by one or more other actuators in the system, or by a large disturbance force.

Figure 17 (left) shows the beam strain response to a piezoceramic applied electric field (that is similar in
form) when no lateral force is applied to the beam tip (dashed), and when a large force is applied (full) so that the piezoceramic actuator is pre-stressed. The corresponding input–output graphs are shown on the right-hand side increase of approximately 11.4% in the actuator gain (measured as the minimum to maximum slope) is observed. In each diagram case the large constant strain induced by the applied tip force is removed for the sake of clarity.

The basic Preisach model can be calculated for each case, and the effect of the independent applied stress suggests that a two-input Preisach model should be used.

4.5. SMA tests

For the SMA tests, the controlled parameter is the wire current and the measured parameter is the strain induced in the beam very close to the cantilevered root (the beam hub is fixed rather than pinned for these experiments). Since the SMA amplifier produces a voltage output a 1 Ω current sensing resistor is used to measure the applied current and a two-state, low-pass filter in a negative feedback loop provided stable current command following with a very good step response.

Ideally, temperature would be the controlled input parameter. In the absence of a reliable temperature sensor, the input current is considered the next most appropriate choice assuming sufficient time is given to allow the temperature to stabilize. The steady-state temperature will then be determined by the applied electrical heat (for a given ambient temperature) which in turn is largely determined by the applied current. One source of error in this approach is the hysteretic straining of the wire which will alter its geometry, the surface area to volume ratio, the heat dissipation function, and thus the temperature to applied heat relationship both in the transient and steady-state cases. If one assumes a maximum wire extensional strain on the order of 4% then the corresponding reduction in diameter is approximately −2% and the surface area to volume ratio reduction is also approximately −2%. This will be an upperbound of the induced error. Another reason for error is the long time constants associated with the heating and cooling processes and this effect can be seen to widen the measured hysteretic loops when the commanded current rate of change is too large. For this reason, a very slow rate of change was used. A third source of
Figure 15. PZT experimental versus simulated: outputs (left), I/O graphs (right).

Figure 16. PZT compensated versus uncompensated; inputs (left), I/O graphs (right).

Figure 17. PZT independent stress comparison: outputs (left), I/O graphs (right).

error is the fluctuating environmental convection currents and temperature. In an effort to minimize this effect, each experiment was repeated several times (nine in all) and the measured signals averaged.

The command trajectories were not smooth as in the piezoceramic case, rather piecewise linear signals were applied, as the integral heating effect provides the necessary smoothing.
Figure 18. SMA minor-loop test: input current (left), output strain (right).

Figure 19. SMA minor-loop test: I/O graph.

4.5.1. Minor-loop tests. Figure 18 (left) shows the input current command used to generate two comparable minor loops with an intervening dominant maxima. The first minor-loop input is traced from points 4 to 6 while the second one is traced from points 11 to 13. Note the slow rate of change of the commanded current.

Figure 18 (right) displays the resulting strain response near the root of the beam and the second minor-loop response is seen to be significantly higher than the first. Note the relatively high rate of change in the output between points 3 and 7, and between points 9 and 14 due to the speed with which the martensite/austenite transformations take place.

Figure 19 is of the corresponding input–output graph and the comparable minor loops can be seen to display a high degree of congruency. The data, however, are still significantly corrupted by thermal transfer fluctuations. Furthermore, for this 0.008" diameter SMA wire the cooling rate is slower than the heating rate for a given step change in current, and further tests are required at an even slower cooling speed.

Note that unlike the piezoceramic cases a major loop may be closely achieved since the temperature of the wire may be raised until the full austenite state is reached, and lowered to the environmental temperature (20°C approximately) so that the full martensite state is almost reached. The evidence for this is the strong ‘knees’ present at the start and end of the transition regions (indeed they are often used to define this region), and the convergence of the increasing and decreasing curves for the high and low current levels, particularly in the former case.

4.5.2. Wiping-out tests. Figure 20 (left) shows the commanded current for the wiping-out test and this to a large extent mirrors that used for the piezoceramic case. Only six dominant extrema are generated before the signal repeats itself and then falls back to zero. Again, piecewise linear segments replace smooth sinusoid components.

The beam strain response is shown in figure 20 (right) (the average over nine experiments) and the second set of extrema are seen to approximately match the first set indicating that the effect of the former is at least partially wiped out by the first dominant maxima of the latter. Figure 21 is the corresponding input–output graph and a large degree of congruency is seen between the first and second half of the experiment with the exception of the initial rising and falling segments.

4.5.3. Conclusions. This paper presents the application of the Preisach hysteresis model to the analysis of hysteresis found in piezoceramics and shape memory alloys. The basic properties of the Preisach model is reviewed, and comparison of experimental input–output responses of PZT and SMA with the Preisach model assumptions, minor-loop congruency and wiping-out properties, is presented.

The piezoceramic tests showed a very high degree of congruency for the comparable minor loops, and the wiping-out property was largely satisfied for the decaying sinusoid experiment. These results suggest that the Preisach model would be suitable for modeling
Preisach modeling of piezoceramic and shape memory alloy hysteresis

Figure 20. SMA wiping-out test: input current (left), output strain (right).

Figure 21. SMA wiping-out test: I/O graph.

the piezoceramic hysteresis and comparisons between simulations and measurements then confirm this conclusion. Furthermore, since the Preisach model has a compact numerical form one now has a means to estimate the effects of hysteresis on specific control strategies before implementation. The Preisach model also has a similarly compact inverse numerical form that may be used as a real-time linearizing compensator that largely cancels the effects of the piezoceramic hysteresis.

The slight difference between vertical chords for congruent minor loops separated by an intervening dominant extremum suggests that a restricted Preisach model (see [5]) might provide a slightly better match. Also, the large gain variation shown by the independent stress test might be modeled by a two-input Preisach model. Further tests are also required to investigate a longer range of test cycles, and to quantify and creep or drift effects.

The SMA minor-loop test also demonstrated a high degree of congruency, even though it was not carried out under a uniform stress condition. Furthermore, the wiping-out test indicated that the wiping-out property holds to a significant extent, and so again a Preisach model would seem appropriate. SMA hysteresis is very pronounced and such a model would be very useful for control strategy analysis, and for hysteresis linearization.

A problem with current tests is that thermal delay/disturbance effects mask the finer detail. Instead of controlling the wire current directly one might apply a high gain/switching beam strain controller instead. If the beam strain can be held approximately constant, then the average wire temperature and therefore current should be constant to a similar extent. Then the strain would be ramped up and down in first-order descending curve experiments, rather the current, but the input-output graph would still contain the same information. The advantage of this approach is that a high gain controller results in correspondingly small heating/cooling delay times and so thermal delay effects are minimized.

However, thermal disturbance effects are now expressed in the current data rather than the strain data and so are not eliminated. A second modification would be to replace one or more short sections of the SMA wire with temperature sensing high tensile strength wires of the same diameter as the SMA. These could be held at a constant temperature above the ambient value and used to estimate the cooling effect of air convection currents. Knowledge of these would then in principle allow for correction of the thermal disturbance effects on the wire current data.

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References


